A Closed-Loop Approach to Continuous Process Scheduling

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Outline

1. Process scheduling problem

2. Decomposition into planning and scheduling
   - Operations planning problem
   - Operations scheduling problem

3. Closed-loop approach
   - Basic idea
   - Operations re-planning model
   - Performance analysis

4. Conclusions
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Problem definition I

Equipment

- Multistage continuous multiproduct production plant
- Multipurpose processing units $u \in U$ operated in continuous mode
- Dedicated storage facilities $s \in S$ of limited capacity
Problem definition II

Operations and states

- Final products arise from sequences of operations executed on dedicated processing units
- During execution of operation materials continuously flow through processing unit
- Each operation $i \in \mathcal{O}$ transforms input states $s \in S_i^-$ into output states $s \in S_i^+$
- Sequence-dependent cleaning times $\vartheta_{ij}$ on processing units
- Processing times $\pi_i$, production rates $\gamma_i$, input and output proportions $\alpha_{is}$ (operating conditions), and start times $\sigma_i$ subject to decision
Problem definition III

Continuous process scheduling problem

Determine production schedule

- operating conditions of operations
- start times of operations

such that

- prescribed bounds for operating conditions are observed
- given primary requirements for final products are satisfied
- no processing unit processes more than one operation at a time
- processing units can be cleaned between consecutive operations
- sufficient amount of input states and sufficient storage space for output states are available during the execution of each operation
- schedule length does not exceed planning horizon
- objective function is optimized (makespan, tardiness, profit)
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Operations planning problem

Determine **operating conditions** of operations subject to
- bounds for operating conditions
- constraints on final inventory levels
- constraints anticipating storage-capacity restrictions

Operations scheduling problem

Determine **start times** of operations subject to
- limited availability of processing units, input states, and storage space for output products
- sequence-dependent cleaning times
- upper bound on schedule length
Basic NLP planning model

Model \((OP)\)

Min. \(f^p(\alpha, \gamma, \pi)\)

\[
\begin{align*}
\text{s.t.} \quad & \sum_{s \in S^i} \alpha_{is} = - \sum_{s \in S^i} \alpha_{is} = 1 \quad (i \in O) \\
& \underline{\alpha}_{is} \leq \alpha_{is} \leq \overline{\alpha}_{is} \quad (i \in O; s \in S^i) \\
& \underline{\gamma}_i \leq \gamma_i \leq \overline{\gamma}_i \quad (i \in O) \\
& \underline{\pi}_i \leq \pi_i \leq \overline{\pi}_i \quad (i \in O) \\
& \delta_s \leq \rho_s^0 + \sum_{i \in O^s} \alpha_{is} \gamma_i \pi_i \leq \overline{\rho}_s \quad (s \in S) \\
& \alpha_{is} \gamma_i = -\alpha_{js} \gamma_j \quad (s \in S; i \in O^{s^+}; j \in O^{s^-}) \\
& \alpha_{is} \gamma_i \max_{j, k \in O^{s^-}} \vartheta_{jk} \leq \overline{\rho}_s \quad (s \in S: \overline{\rho}_s > 0; i \in O^{s^+})
\end{align*}
\]
Basic NLP planning model

Model \((OP)\)

\[
\text{Min. } \tilde{f}^p(\alpha, \gamma, \pi, \zeta) = \|\zeta^1 + \zeta^2\|_1 + \varepsilon f^p(\alpha, \gamma, \pi)
\]

\[
s.t. \quad \sum_{s \in S^i^+} \alpha_{is} = -\sum_{s \in S^i^-} \alpha_{is} = 1 \quad (i \in O)
\]

\[
\alpha_{is} \leq \alpha_{is} \leq \overline{\alpha}_{is} \quad (i \in O; s \in S^i)
\]

\[
\gamma_i \leq \gamma_i \leq \overline{\gamma}_i \quad (i \in O)
\]

\[
\pi_i \leq \pi_i \leq \overline{\pi}_i \quad (i \in O)
\]

\[
\delta_s \leq \rho_s^0 + \sum_{i \in O^s} \alpha_{is} \gamma_i \pi_i \leq \overline{\rho}_s \quad (s \in S)
\]

\[
\alpha_{is} \gamma_i = -\alpha_{js} \gamma_j + \zeta^1_{ijs} - \zeta^2_{ijs} \quad (s \in S; i \in O^{s^+}; j \in O^{s^-})
\]

\[
\alpha_{is} \gamma_i \max_{j, k \in O^{s^-}} \psi_{jk} \leq \overline{\rho}_s \quad (s \in S: \rho_s > 0; i \in O^{s^+})
\]

\[
\zeta^1_{ijs}, \zeta^2_{ijs} \geq 0 \quad (s \in S; i \in O^{s^+}; j \in O^{s^-})
\]
Example (Make-and-mix plant)

Operating conditions for $f^p(\alpha, \gamma, \pi) = \pi_1 + \pi_2 + \pi_3$

- $i = 1$: $\gamma_1 = 0.83$  $\pi_1 = 100.0$  $\alpha = 0.6$
- $i = 2$: $\gamma_2 = 0.5$  $\pi_2 = 33.3$
- $i = 3$: $\gamma_3 = 1.0$  $\pi_3 = 100.0$
Operations scheduling problem I

Solution procedures

- Exact MILP model \( OS(\alpha, \gamma, \pi) \): Herrmann S, S. (2008)

Proposition (Feasibility of scheduling problem)

*If all material flows are acyclic and \( \zeta^1 + \zeta^2 = 0 \), then there exists a feasible solution to the operations scheduling problem, which can be obtained in polynomial time.*
Operations scheduling problem II

Example: optimal schedule for $f(\alpha, \gamma, \pi, \sigma) = \max_{i \in O}(\sigma_i + \pi_i) = C_{\text{max}}$

U2

U1

$0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150$
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Basic idea 1

Motivation

- Maximum inventory levels depend on operations sequence
- To facilitate generation of feasible schedule, planning model (OP) aligns rates $|\alpha_is\gamma_i|$ of producing and consuming operations
- Production rates generally unnecessarily small
Basic idea II

Basic idea

- **Return to planning phase** after scheduling
- **Fix sequence of start and completion events** and replace alignment of rates by exact material-availability and storage-capacity constraints in planning problem
- Inventory levels at support points can be expressed as sums of amounts produced and consumed by **active sets** (antichains)
- Associate **decision variable** $\pi_A$ with each active set $A$ providing its duration
- Processing times $\pi_i$ and start times $\sigma_i$ uniquely given by sequence and durations of active sets
Active sets I

Example (cont’d)

\[ B = \{A_1, \ldots, A_\nu\} = \{\emptyset_1, \{1\}, \{1,3\}, \{3\}_1, \{2,3\}, \{3\}_2, \emptyset_2\} \]

\[ \pi_1 = \pi\{1\} + \pi\{1,3\}, \quad \pi_2 = \pi\{2,3\}, \quad \pi_3 = \pi\{1,3\} + \pi\{3\}_1 + \pi\{2,3\} + \pi\{3\}_2 \]

\[ \sigma_1 = \pi\emptyset_1, \quad \sigma_2 = \pi\emptyset_1 + \pi\{1\} + \pi\{1,3\} + \pi\{3\}_1, \quad \sigma_3 = \pi\emptyset_1 + \pi\{1\} \]
Active sets II

Example (cont’d)

\[
\gamma_1 \pi \{1\} + (\alpha_{14} \gamma_1 + \alpha_{34} \gamma_3) \pi \{1,3\}
\]

\[
\gamma_1 \pi \{1\}
\]
Operations re-planning model

Model \((ORP(\sigma'))\)

\[
\begin{align*}
\text{Min. } & f(\alpha, \gamma, \pi, \sigma) \\
\text{s.t. } & \sum_{s \in S^{i+}} \alpha_{is} = - \sum_{s \in S^{i-}} \alpha_{is} = 1 \quad (i \in O) \\
& \alpha_{is} \leq \alpha_{is} \leq \overline{\alpha}_{is} \quad (i \in O; s \in S^i) \\
& \gamma_i \leq \gamma_i \leq \overline{\gamma}_i \quad (i \in O) \\
& \pi_i \leq \pi_i \leq \overline{\pi}_i \quad (i \in O) \\
& \delta_s \leq \rho_0^s + \sum_{i \in O^s} \alpha_{is} \gamma_i \pi_i \leq \overline{\rho}_s \quad (s \in S) \\
& \pi_i = \sum_{A \in B_s^s(\nu_s) : i \in A} \pi_A \quad (i \in O; s \in S^i) \\
& \sigma_i = \sum_{A \in B_s^s(\mu)} \pi_A \quad (s \in S; \mu = 1, \ldots, \nu^s - 1; i \in A^{s+1}_\mu \setminus A^s_\mu) \\
& 0 \leq \rho_0^s + \sum_{A \in B_s^s(\mu)} \sum_{i \in A} \alpha_{is} \gamma_i \pi_A \leq \overline{\rho}_s \quad (s \in S; \mu = 1, \ldots, \nu^s - 1) \\
& \sigma_j - \sigma_i \geq \pi_i + \vartheta_{ij} \quad (u \in U; \ i, j \in O^u : \sigma'_j \geq \sigma'_i) \\
& \sigma_i + \pi_i \leq \tau \quad (i \in O) \\
& \pi_A \geq 0 \quad (s \in S; A \in B^s(\nu_s))
\end{align*}
\]
Closed-loop method I

Closed-loop method

**Input:** process scheduling problem

**Output:** feasible production schedule \((\alpha, \gamma, \pi, \sigma')\)

determine initial operating conditions \((\alpha, \gamma, \pi)\) by solving basic planning model \((OP)\);

repeat

compute schedule \(\sigma'\) by solving resulting scheduling problem \((OS(\alpha, \gamma, \pi))\);

re-optimize operating conditions \((\alpha, \gamma, \pi)\) with operations re-planning planning model \((ORP(\sigma'))\);

until fixed point \((\alpha, \gamma, \pi, \sigma')\) has been reached;

return feasible production schedule \((\alpha, \gamma, \pi, \sigma')\);
Proposition (Monotonicity and finiteness)

Provided that problems (OP) and (OS(\(\alpha, \gamma, \pi\))) are feasible, the sequence of generated objective function values \(f(\alpha, \gamma, \pi, \sigma')\) is nonincreasing. The closed-loop method attains a fixed point after a finite number of iterations.

Proof.

The monotonicity follows from the feasibility of the preceding production schedule \((\alpha, \gamma, \pi, \sigma')\) with respect to \((ORP(\sigma'))\). The feasible region of \((ORP(\sigma'))\) only depends on the sequence of active sets \(A_\mu\) induced by \(\sigma'\). In conjunction with the monotonicity this provides the finiteness.
Example (cont’d)

Initial schedule after scheduling

\[
\begin{align*}
\gamma_1 &= 0.83 \\
\alpha &= 0.6 \\
\gamma_2 &= 0.5 \\
\gamma_3 &= 1.0 \\
C_{\text{max}} &= 143.3
\end{align*}
\]
Example (cont’d)

Second schedule after re-planning

<table>
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<tr>
<th>U2</th>
<th>U1</th>
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<tbody>
<tr>
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<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

γ₁ = 1.0
α = 0.6
γ₂ = 2.0
γ₃ = 1.54
Cₘₐₓ = 115.0
Example (cont’d)

Third schedule after scheduling

\[ \gamma_1 = 1.0 \]
\[ \alpha = 0.6 \]
\[ \gamma_2 = 2.0 \]
\[ \gamma_3 = 1.54 \]
\[ C_{\text{max}} = 101.7 \]
Example (cont’d)

Fourth and final schedule after re-planning / scheduling

\[ \gamma_1 = 1.0 \]
\[ \alpha = 0.71 \]
\[ \gamma_2 = 2.0 \]
\[ \gamma_3 = 1.43 \]
\[ C_{\text{max}} = 95.0 \]
Performance analysis I

Test bed: FMCG case study

- Case study from FMCG industry (Méndez and Cerdá 2002)
- Objective makespan minimization
- 10 instances with varying primary requirements for final products
- Planning models solved under GAMS using NLP solver CONOPT3
- Scheduling model solved under GAMS using MILP solver Cplex 11.0
- Lower bounds computed with (relaxation of) MILP model by Méndez and Cerdá 2002, time limit 3600.0 sec
- Pentium IV PC, 3.8 GHz, 2 GB RAM
Performance analysis II

<table>
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<tr>
<th>$d_1$–$d_5$</th>
<th>$d_6$–$d_{10}$</th>
<th>$d_{11}$–$d_{15}$</th>
<th>$C_{\text{max}}^{\text{ini}}$</th>
<th>$C_{\text{max}}^{\text{fin}}$</th>
<th>$n_{\text{it}}$</th>
<th>$t_{\text{cpu}}$ [sec]</th>
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<td>192.6</td>
<td>4</td>
<td>46.4</td>
<td>188.6</td>
</tr>
</tbody>
</table>
Performance analysis III

Test bed: Kallrath case study

- Case study of Kallrath 2002 adapted to continuous production mode
- Objective makespan minimization
- 8 instances with varying primary requirements for final products
- Planning models solved under GAMS using NLP solver CONOPT3
- Scheduling model solved under GAMS using MILP solver Cplex 11.0
- Lower bounds computed with (tightened version of) MILP model by Giannelos and Georgiadis 2002, time limit 3600.0 sec
- Pentium IV PC, 3.8 GHz, 2 GB RAM
## Performance analysis IV

### Computational results: Kallrath case study

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<tr>
<th>(d_{15})</th>
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<th>(d_{17})</th>
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<th>(d_{19})</th>
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<th>(C_{\text{max}}^{\text{fin}})</th>
<th>(n_{\text{it}})</th>
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<th>(C_{\text{max}}^{\text{milp}})</th>
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Summary

- **Decomposition approach** for process scheduling of continuous multiproduct plants
- **Planning problem**: determine operating conditions of operations
- **Scheduling problem**: schedule operations on processing units
- **Closed-loop method**: re-optimize operating conditions subject to constraints on active sets
- **Fixed point** reached in finite number of iterations
- **Good schedules within reasonable amount of time**, high **reliability**

Future research

- Tests for alternative objective functions (revenues, profit, tardiness)
- Metaheuristic search procedure performing perturbation steps after convergence
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European Journal of Operational Research 165:495–509
Backup: The scheduling model

**Model \((OS(\alpha, \gamma, \pi))\)**

\[
\text{Min. } f(\alpha, \gamma, \pi, \sigma)
\]
\[
\text{s.t. } 0 \leq \sigma_i \leq \tau - \pi_i
\]
\[
\pi_i + \vartheta_{ij} - \tau (1 - z_{ij}) \leq \sigma_j - \sigma_i \leq -\pi_j - \vartheta_{ji} + \tau z_{ij}
\]
\[
0 \leq x_{ijs} \leq 1
\]
\[
x_{ijs} \geq y_{ijs}
\]
\[
x_{ijs} \leq 1 - y_{ijs}
\]
\[
\pi_i - \tau (1 - y_{ijs}) \leq \sigma_j + \pi_j - \sigma_i \leq \pi_i x_{ijs} + \tau y_{ijs}
\]
\[
\pi_i - \tau (1 - y_{ijs}) \leq \sigma_j - \sigma_i \leq \pi_i x_{ijs} + \tau y_{ijs}
\]
\[
\pi_i x_{ijs} - \tau y_{ijs} \leq \sigma_j + \pi_j - \sigma_i \leq \tau (1 - y_{ijs})
\]
\[
\pi_i x_{ijs} - \tau y_{ijs} \leq \sigma_j - \sigma_i \leq \tau (1 - y_{ijs})
\]
\[
\rho_s^0 + \sum_{i \in O^s} \alpha_i \gamma_i \pi_i x_{ijs} \geq 0
\]
\[
y_{ijs} \in \{0, 1\}
\]
\[
z_{ij} \in \{0, 1\}
\]