SCHEDULING CONTINUOUS MATERIAL FLOWS

Christoph Schwindt, Norbert Trautmann
Institut für Wirtschaftstheorie und Operations Research,
University of Karlsruhe, 76128 Karlsruhe, Germany.
{schwindt,trautmann}@wior.uni-karlsruhe.de

In process industries, final products result from several successive chemical or physical transformations of raw materials using scarce production resources. We consider the scheduling of a production plant processing a given set of operations in continuous production mode. The different production levels are decoupled by intermediate storage facilities. For each operation, the processing time, the production resources used, and the total quantities of input and output products consumed and produced, respectively, are known. The problem consists of allocating the resources over time such that during the processing of each operation, a sufficient amount of input products and sufficient storage capacity for output products is available, prescribed time lags between operations are observed, and some convex objective function in the start times of the operations is minimized. For solving this problem, we propose a branch-and-bound algorithm that is based on the representation of resource and inventory constraints as disjunctions of linear inequality systems.

1 INTRODUCTION

Let $\mathcal{O}$ be a set of operations to be executed on a chemical production plant driven in continuous production mode. The production plant consists of processing units that are linked by storage facilities for intermediate products. Each operation $i \in \mathcal{O}$ is associated with a processing time p_i. While being in progress, i consumes input and produces output products at constant rates and requires r_{ik} units of certain production resources k with capacity R_k such as processing units and operators. Let $\mathcal{R}$ be the set of all resources used and let $\mathcal{P}$ be the set of all products consumed and produced by operations $i \in \mathcal{O}$. $\mathcal{O}_\pi^-$ and $\mathcal{O}_\pi^+$ are the sets of operations consuming or producing, respectively, product $\pi \in \mathcal{P}$. For $i \in \mathcal{O}_\pi^-$, $\rho_{i\pi} < 0$ denotes the negative amount of product π consumed and for $i \in \mathcal{O}_\pi^+$, $\rho_{i\pi} > 0$ denotes the (positive) amount of product π produced by operation i. Each product $\pi \in \mathcal{P}$ is stocked in a dedicated storage facility with storage capacity $\overline{R}_\pi \geq 0$. $\underline{R}_\pi \leq \overline{R}_\pi$ is the safety stock of product π. For certain pairs of operations $(i,j) \in E \subset \mathcal{O} \times \mathcal{O}$, a minimum time lag δ_{ij} between the start of operation i and the start of operation j is prescribed. A negative minimum time lag δ_{ij} can be interpreted as positive maximum time lag $-\delta_{ij}$ between operation j and operation i. These temporal constraints may arise from technological or organizational requirements such as release dates for raw materials, delivery dates for final products, or precedence relationships between operations.

We consider the problem of finding a start time S_i for each operation $i \in \mathcal{O}$ such that some convex function $f(S_1, \ldots, S_n)$ like the production makespan $\max_{i\in\mathcal{O}}(S_i + p_i)$ is minimized and the temporal constraints as well as the resource and inventory constraints arising from the limited availability of resources, storage capacities, and input products are taken into account.

The problem of minimizing the makespan subject to resource constraints and temporal constraints between operations has been treated in the framework of resource-constrained project scheduling, see for example Bartusch et al. (1988), De Reyck and Herroelen (1998), Dorndorf et al. (2000), or Franck et al. (2001). Neumann and Schwindt (1999) and Laborie (2001) have considered project scheduling problems subject to inventory constraints for the case of infinite production and consumption rates. The scheduling of batch production plants in process industries including production resources and storage facilities has been dealt with in Schwindt and Trautmann (2000).

2 MODEL

Let $S = (S_1, \ldots, S_n)$ with $S_i \geq 0$ for all $i \in \mathcal{O}$ be some production schedule. Then $\mathcal{A}(S,t) := \{i \in \mathcal{O} \mid S_i \leq t < S_i + p_i\}$ is the set of activities being processed at time t and

$$r_k(S,t) := \sum_{i\in\mathcal{A}(S,t)} r_{ik}$$

is the amount of resource $k \in \mathcal{R}$ used at time t. By

$$x_i(S,t) = \begin{cases} 0, \text{ if } t < S_i \\ 1, \text{ if } t \geq S_i + p_i \\ (t - S_i)/p_i, \text{ otherwise} \end{cases}$$

we denote the portion of operation $i \in \mathcal{O}$ that has been processed by time t. The inventory of product $\pi \in \mathcal{P}$ at time t is

$$\rho_\pi(S,t) := \sum_{i\in\mathcal{O}_\pi} \rho_{i\pi} x_i(S,t)$$

where $\mathcal{O}_\pi := \mathcal{O}_\pi^- \cup \mathcal{O}_\pi^+$. The temporal constraints can be written as

$$S_j - S_i \geq \delta_{ij} \quad ((i,j) \in E)$$

The production scheduling problem to be dealt with can now be formulated as follows.

$$\begin{array}{lllr} \text{Minimize} & f(S) & & \\ \text{subject to} & r_k(S,t) \leq R_k & (k \in \mathcal{R},\ t \geq 0) & (1) \\ & \underline{R}_\pi \leq \rho_\pi(S,t) \leq \overline{R}_\pi & (\pi \in \mathcal{P},\ t \geq 0) & (2) \\ & S_j - S_i \geq \delta_{ij} & ((i,j) \in E) & (3) \\ & S_i \geq 0 & (i \in \mathcal{O}) & (4) \end{array}$$

Let $\mathcal{S}_R$ and $\mathcal{S}_I$ denote the sets of schedules complying with the resource constraints (1) and the inventory constraints (2), respectively. By $\mathcal{S}_T$ we denote the set of all schedules satisfying the temporal constraints (3). $\mathcal{S} = \mathcal{S}_R \cap \mathcal{S}_I \cap \mathcal{S}_T$ is the set of all feasible schedules. An optimal schedule is a feasible schedule minimizing f.

3 SOLUTION METHOD

In what follows, we extend an approach devised by De Reyck and Herroelen (1998) for project scheduling with renewable resources to deal with our problem. Neumann and Schwindt (1999) have generalized the latter approach to scheduling problems with inventory constraints where the consumption and production rates are infinite.

The basic principle is as follows. We omit the difficult resource and inventory constraints (1) and (2) and solve the resulting convex program by computing some schedule S that minimizes function f on polyhedron $\mathcal{S}_T$. If S is feasible, S is an optimal schedule. Otherwise, we determine some point in time t for which constraint (1) or (2) is not satisfied.

Excessive utilization of some resource $k \in \mathcal{R}$ at time t can be resolved by finding a partition (A, B) of $\mathcal{A}(S, t)$ such that A is an $\subseteq$-maximal set with total requirements $\sum_{i \in A} r_{ik} \leq R_k$. We then select some operation i from set A and introduce precedence relationships between i and all operations $j \in B$, that is, we add the temporal constraints

$$S_j - S_i \geq p_i \quad (j \in B) \tag{5}$$

to the convex program, which is then solved again.

We now consider how to deal with violations of the inventory constraints. For notational convenience, we suppose that all storage capacities are infinite. This can always be ensured by the following transformation. For each product $\pi \in \mathcal{P}$, we set $\overline{R}_\pi := \infty$ and add a fictitious product π' with $\rho_{i\pi'} := -\rho_{i\pi}$ for all $i \in \mathcal{O}_\pi$ and $\underline{R}_{\pi'} := -\overline{R}_\pi$, $\overline{R}_{\pi'} := \infty$.

Now assume that at time t, the inventory of some product $\pi \in \mathcal{P}$ falls below the safety stock $\underline{R}_\pi$. We partition $\mathcal{O}_\pi$ into two sets A and B with the following meaning. A contains all operations $j \in \mathcal{O}_\pi^-$ that will be completed by deadline t and all operations $j \in \mathcal{O}_\pi^+$ that will be released at time t, that is,

$$\left.\begin{array}{ll} S_j \leq t - p_j & (j \in A \cap \mathcal{O}_\pi^-) \\ S_j \geq t & (j \in A \cap \mathcal{O}_\pi^+) \end{array}\right\} \tag{6}$$

The demand for product π at time t arising from operations $j \in A$ equals $-\sum_{j \in A \cap \mathcal{O}_\pi^-} \rho_{j\pi}$. The operations j from set B must be scheduled such that at time t, their replenishment of product π is greater than or equal to the inventory shortage caused by the operations from set A, i.e., $\sum_{j \in B} \rho_{j\pi} x_j(S, t) \geq \underline{R}_\pi - \sum_{j \in A \cap \mathcal{O}_\pi^-} \rho_{j\pi}$. This can be ensured as follows. For each operation $j \in B$, we introduce a continuous decision variable x_j with

$$0 \leq x_j \leq 1 \quad (j \in B) \tag{7}$$

providing the portion of operation j that will be processed by time t. The requirement that the inventory of product π at time t must not fall below $\underline{R}_\pi$ then reads

$$\sum_{j\in B} \rho_{j\pi} x_j \geq \underline{R}_\pi - \sum_{j\in A\cap\mathcal{O}_\pi^-} \rho_{j\pi} \tag{8}$$

The coupling between decision variables x_j and S_j is achieved by the release date and deadline constraints (parameterized in x_j)

$$\left.\begin{array}{ll} S_j \geq t - p_j x_j & (j \in B \cap \mathcal{O}_\pi^-) \\ S_j \leq t - p_j x_j & (j \in B \cap \mathcal{O}_\pi^+) \end{array}\right\} \tag{9}$$

Inequalities (9) ensure that for each schedule S satisfying (9), $x_j \geq x_j(S,t)$ if operation $j \in B$ depletes and that $x_j \leq x_j(S,t)$ if operation $j \in B$ replenishes the stock of π. Adding constraints (6) to (9) to the convex program settles the inventory shortage at time t.

The inventory of a product π attains its minimum at a point in time t when some producing operation i is started or when some consuming operation i is terminated. That is why time t can always be chosen to be equal to S_i for some $i \in \mathcal{O}_\pi^+$ or equal to $S_i + p_i$ for some $i \in \mathcal{O}_\pi^-$, and thus we can replace t in (6) and (9) by S_i or $S_i + p_i$. We then write $A^{\pi i}$ and $B^{\pi i}$ instead of A and B as well as $x_j^{\pi i}$ instead of x_j. Note that without loss of generality we can assume $i \in A^{\pi i}$ for all $\pi \in \mathcal{P}$ and all $i \in \mathcal{O}_\pi$ because the corresponding inequality (6) is always satisfied. Replacing constants t with variables S_i ensures that only a finite number of constraints have to be introduced before the inventory constraints are satisfied.

From the above reasoning we obtain the following Lemma.

Lemma 1 *A schedule S is inventory-feasible if and only if for each product $\pi \in \mathcal{P}$ and each operation $i \in \mathcal{O}_\pi^+$ ($i \in \mathcal{O}_\pi^-$) there exists a partition $\{A^{\pi i}, B^{\pi i}\}$ of set $\mathcal{O}_\pi$ such that for some $x^{\pi i} \in [0,1]^{|B^{\pi i}|}$*

1. *$S_i - S_j \geq p_j$ ($p_j - p_i$) for all $j \in \mathcal{O}_\pi^- \cap A^{\pi i}$,*
2. *$S_j - S_i \geq 0$ (p_i) for all $j \in \mathcal{O}_\pi^+ \cap A^{\pi i}$,*
3. *$S_j - S_i \geq -p_j x_j^{\pi i}$ ($p_i - p_j x_j^{\pi i}$) for all $j \in \mathcal{O}_\pi^- \cap B^{\pi i}$,*
4. *$S_i - S_j \geq p_j x_j^{\pi i}$ ($p_j x_j^{\pi i} - p_i$) for all $j \in \mathcal{O}_\pi^+ \cap B^{\pi i}$, and*
5. *$\sum_{j\in B^{\pi i}} \rho_{j\pi} x_j^{\pi i} \geq \underline{R}_\pi - \sum_{j\in A^{\pi i}\cap\mathcal{O}_\pi^-} \rho_{j\pi}$.*

In Bartusch et al. (1988) it has been shown that set $\mathcal{S}_R$ represents the union of finitely many polyhedra each of which possesses a unique minimal point. As a direct consequence of Lemma 1, we obtain the following characterization of the set of inventory-feasible schedules $\mathcal{S}_I$.

Proposition 2 *$\mathcal{S}_I$ represents the union of finitely many polyhedra. The set of all minimal points of $\mathcal{S}_I$ is generally uncountable.*

The solution procedure for the production scheduling problem is now as follows. We solve the convex program

$$\text{(CP)}\quad \begin{cases} \text{Minimize} & f(S) \\ \text{subject to} & (3) \text{ and } (4) \\ & (5) \text{ for pairs } (i,B) \text{ selected} \\ & (6) \text{ to } (9) \text{ for partitions } \{A^{\pi i}, B^{\pi i}\} \text{ selected} \end{cases}$$

—at the beginning, no pairs (i, B) and no partitions $\{A^{\pi i}, B^{\pi i}\}$ have been selected—and add new constraints of type (5) or (6) to (9) either until the feasible region of (CP) becomes void or until the resulting schedule S is feasible. Then we return to an alternative pair (i, B) or an alternative partition $\{A^{\pi i}, B^{\pi i}\}$ and proceed until all alternatives have been investigated. Since its feasible region represents a polyhedron, (CP) can be solved in polynomial time by the ellipsoid method (cf. Grötschel et al., 1995) or some specific barrier method (cf. Nesterov and Nemiroskii, 1994). The objective function value of any optimal solution to (CP) represents a lower bound on the objective function value $f(S)$ of any feasible schedule satisfying the added constraints of type (5) to (9).

Example 3 We consider an example with one intermediate product π and three operations $i = 1, 2, 3$ executed on processing units U_1, U_2, and U_3, respectively (cf. Figure 1). Operation 1 with processing time $p_1 = 3$ produces $\rho_{1\pi} = 2$ units of product π, which are consumed by operations 2 and 3 with processing times $p_2 = p_3 = 2$ and requirements $\rho_{2\pi} = \rho_{3\pi} = -1$. The minimum inventory of product π is $\underline{R}_\pi = 0$. For simplicity we suppose that a sufficient amount of raw material is available, that no operators are needed, and that there are no temporal constraints given (i.e., $E = \emptyset$). Since each processing unit only processes one operation, production resources may be disregarded, i.e., $\mathcal{R} = \emptyset$. The objective function to be minimized is the makespan.

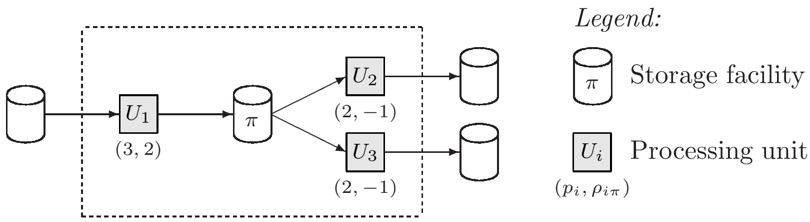

Figure 1 – Process flow chart

Solving the initial convex program (CP) provides the earliest schedule S, where all operations are started at time 0. Figure 2 shows the execution time intervals of operations $i \in \mathcal{O}$ and the inventory profile $\rho_\pi(S, \cdot)$ over time for product π. Schedule S is not inventory-feasible because in interval $]0, 3[$ the inventory of product π is negative.

One way to settle the inventory shortage at the completion time of operation $i = 2$ is to choose $A^{\pi 2} = \{2\}$ and $B^{\pi 2} = \{1, 3\}$, and thus we add the constraints

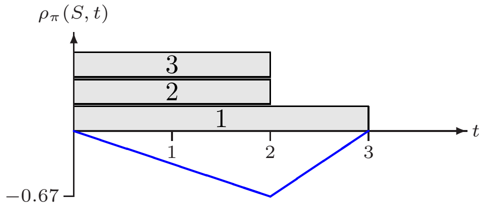

Figure 2 – Inventory profile arising from relaxing inventory constraints

$0 \leq x_1^{\pi 2} \leq 1$, $0 \leq x_3^{\pi 2} \leq 1$, $S_2 - S_1 \geq 3x_1^{\pi 2} - 2$, $S_3 - S_2 \geq 2 - 2x_3^{\pi 2}$, and $2x_1^{\pi 2} - x_3^{\pi 2} \geq 1$ to (CP). An optimal solution to the resulting convex program is $x_1^{\pi 2} = 1$, $x_3^{\pi 2} = 1$, $S_1 = 0$, $S_2 = 1$, $S_3 = 1$. Figure 3 shows the execution time intervals and the inventory profile for the new schedule S. Since $\rho_\pi(S,t) \geq 0$ for all $t \geq 0$ and $f(S) = 3 = p_1$, schedule S is optimal.

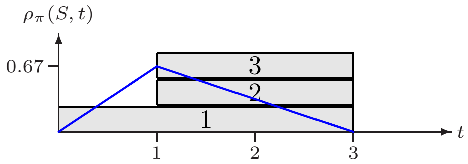

Figure 3 – Inventory profile after addition of constraints (6) to (9)

Figure 4 shows the gray-shaded projection of the set $\mathcal{S}$ of all feasible schedules onto the two-dimensional subspace of $\mathbb{Q}^3$ with $S_1 = 0$. The polyhedron bounded by the dashed-line edges corresponds to the feasible region of the convex program for $A^{\pi 2} = \{2\}$ and $B^{\pi 2} = \{1,3\}$. Optimal schedule S is marked by a solid circle. The line segments joining points $(0, 1.33)$ and $(1,1)$ and points $(1,1)$ and $(1.33, 0)$ contain all minimal points of $\mathcal{S}$.

4 IMPLEMENTATION ISSUES

Assume that the inventory of some product $\pi \in \mathcal{P}$ falls below the safety stock at time $t = S_i$ ($i \in \mathcal{O}_\pi^+$) or $t = S_i + p_i$ ($i \in \mathcal{O}_\pi^-$). To enumerate the sets $A^{\pi i}$ and $B^{\pi i}$ we construct a binary tree as follows. Each level of the tree belongs to one operation $j \in \mathcal{O}_\pi$. For each j, we branch over the alternatives $j \in A^{\pi i}$ and $j \in B^{\pi i}$ and add the corresponding constraints (6) or (7), (9), as well as for both alternatives the relaxation

$$\sum_{j \in B^{\pi i}} \rho_{j\pi} x_j^{\pi i} \geq \underline{R}_\pi - \sum_{j \in A^{\pi i} \cap \mathcal{O}_\pi^-} \rho_{j\pi} - \sum_{j \in \mathcal{O}_\pi^+ \setminus A^{\pi i} \setminus B^{\pi i}} \rho_{j\pi} \tag{10}$$

of constraint (8) to the convex program (CP). Each leaf of the tree corresponds to one distinct partition $\{A^{\pi i}, B^{\pi i}\}$. We can suspend the enumeration for operation i as soon as the inventory shortage at time S_i or $S_i + p_i$ is settled, even if $A^{\pi i} \cup B^{\pi i} \subset \mathcal{O}_\pi$. In the latter case, it may be necessary to resume the branching process later on if the shortage reappears while dealing with

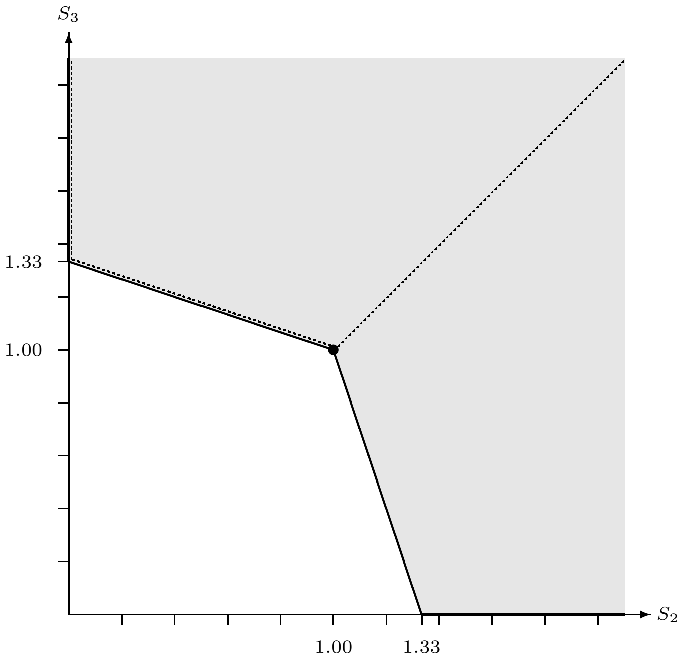

Figure 4 – Projection of the feasible region $\mathcal{S}$

other shortages or violations of the resource constraints (1). Since at most $n(n-1)/2$ precedence constraints $S_j - S_i \geq p_i$ between operations i and j can be defined and because for each $\pi \in \mathcal{P}$ and each $i \in \mathcal{O}_\pi$, the construction of the corresponding sets $A^{\pi i}$ and $B^{\pi i}$ requires at most $|\mathcal{O}_\pi|$ steps, the height of the branch-and-bound tree is of order $O(\max[n^2, \sum_{\pi \in \mathcal{P}} |\mathcal{O}_\pi|^2])$.

The size of the enumeration tree can be reduced by performing immediate selection and applying dominance rules. Let d_{ij} be the minimum time lag between operations i and j that is implied by the prescribed temporal constraints (3) or the added temporal constraints corresponding to the precedence constraints (5), inequalities (6), and inequalities (9), where $x_j^{\pi i}$ is set to be equal to 0 if $j \in \mathcal{O}_\pi^-$ and equal to 1, otherwise. The values d_{ij} can be computed by calculating longest path lengths in a network with node set $\mathcal{O}$ containing an arc (i, j) for each prescribed or added temporal constraint. The arcs are weighted with the respective minimum time lags. Now assume that for some operation $j \in \mathcal{O}_\pi$, the addition to set $A^{\pi i}$ or $B^{\pi i}$, respectively, leads to a new temporal constraint $S_j - S_i \geq \delta_{ij}$. Then the feasible region of (CP) becomes void if $\delta_{ij} + d_{ji} > 0$. In that case, the alternative set $B^{\pi i}$ or $A^{\pi i}$, respectively, can immediately be selected for operation j.

Now let (S, x) be an optimal solution to (CP) such that schedule S is feasible. We then obtain a feasible schedule S' with $f(S') \leq f(S)$ by

1. moving all operations $j \in \mathcal{O}_\pi$ from $A^{\pi i}$ to $B^{\pi i}$ for which (6) is binding,
2. moving all operations $j \in \mathcal{O}_\pi^-$ from $B^{\pi i}$ to $A^{\pi i}$ for which $x_j^{\pi i} = 1$, and
3. moving all operations $j \in \mathcal{O}_\pi^+$ from $B^{\pi i}$ to $A^{\pi i}$ for which $x_j^{\pi i} = 0$.

Based on this dominance rule, feasible solutions belonging to leaves of the enumeration tree can be improved and thus the current upper bounds decreased by performing the above transformations. For the example of Section 3, we obtain the same optimal schedule $S' = S$ by moving operation $3 \in \mathcal{O}_\pi^-$ with $x_3^{\pi 2} = 1$ from set $B^{\pi 2}$ to set $A^{\pi 2}$.

5 CONCLUSIONS

In this paper we have considered the scheduling of a production plant operating in continuous production mode. The problem consists of allocating scarce production resources over time to the processing of a given set of operations subject to temporal constraints arising from technological or organizational requirements and inventory constraints referring to continuous material flows. The objective is to minimize some convex function in the start times of the operations like the production makespan. Proceeding from an analysis of the feasible region, we have developed the enumeration scheme of a relaxation-based branch-and-bound algorithm for this problem. The procedure iteratively substitutes the resource and inventory constraints into a disjunction of linear inequality systems.

Future research in this area will be concerned with the development of local search algorithms based on the concepts presented and the integration of additional constraints that are frequently encountered in practice like sequence-dependent changeover times or alternative recipes.

REFERENCES

1. Bartusch M, Möhring RH, Radermacher, FJ (1988) Scheduling project networks with resource constraints and time windows. Annals of Operations Research 16:201–240
2. De Reyck B, Herroelen WS (1998) A branch-and-bound procedure for the resource-constrained project scheduling problem with generalized precedence relations. European Journal of Operational Research 111:152–174
3. Dorndorf U, Pesch E, Phan Huy T (2000) A time-oriented branch-and-bound algorithm for resource-constrained project scheduling with generalised precedence constraints. Management Science 46:1365–1384
4. Franck B, Neumann K, Schwindt C (2001) Truncated branch-and-bound, schedule-construction, and schedule-improvement procedures for resource-constrained project scheduling. OR Spektrum 23:297–324
5. Grötschel M, Lovász L, Schrijver A (1995) Geometric Algorithms and Combinatorial Optimization. Springer, Berlin
6. Laborie P (2001) Algorithms for propagating resource constraints in AI planning and scheduling: Existing approaches and new results. Proceedings of the IJCAI-01 Workshop on Planning with Resources, Seattle, WA, USA
7. Nesterov Y, Nemirovskii A (1994) Interior-Point Polynomial Algorithms in Convex Programming. SIAM, Philadelphia
8. Neumann K, Schwindt C (1999) Project scheduling with inventory constraints. Report WIOR-572, University of Karlsruhe
9. Schwindt C, Trautmann N (2000) Batch scheduling in process industries: An application of resource-constrained project scheduling. OR Spektrum 22:501–524