STORAGE PROBLEMS IN PROCESS SCHEDULING

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This paper is concerned with the modelling of storage configurations for intermediate products in process industries. Such a model forms the basis of algorithms for scheduling chemical production plants. Different storage capacity settings (unlimited, finite, and no intermediate storage), storage homogeneity settings (dedicated and shared storage), and storage time settings (unlimited, finite, and no wait) are considered. We discuss a classification of storage constraints in process scheduling and show how those constraints can be integrated into a general production scheduling model that is based on the concept of cumulative resources.

1 PROCESS SCHEDULING

In process industries, e.g. the chemical or food industry, final products result from successive chemical or physical transformations of raw materials on scarce production resources (cf. Applequist et al. 1997). The problem under consideration consists of scheduling a given set of operations on a production plant such that a given objective function is minimized. Typical objectives for process scheduling problems are the minimization of makespan, lateness, or setup costs.

Each operation requires a processing unit and operating staff for execution. In addition, each operation consumes a given amount of one or several input products and produces a given amount of one or several output products. Certain intermediate products can then be buffered in storage facilities of given capacity. Other intermediate products cannot be stored and thus must be consumed without any delay. We suppose that for the storable intermediate products, a nonnegative safety stock is prescribed. Since one and the same product may be produced by several operations and may be consumed by several operations, it is generally not possible to establish a-priori precedence relationships between producing and consuming operations. If the plant is operated in batch production mode, i.e., the material flow is discontinuous, the input products of an operation must be available at its start and the output products arise at its completion. Moreover, we assume that at the end of an operation, the output products are unloaded from the processing unit. If the plant is operated in continuous production mode, the products are produced and consumed at constant rates. Production plants are driven in batch production mode if small amounts of a large number of products are required. In this case, the plant is configured according to the required final products. Continuous production mode is typical of mass production like oil refinement.

In chemical engineering literature (cf. e.g. Ha et al. 2000, Kim et al. 1996, or Kim et al. 2000), the following *storage capacity settings* for a given intermediate product are considered:

1. Unlimited intermediate storage UIS: The storage facility has unlimited capacity.
2. Finite intermediate storage FIS: The storage facility has limited capacity only.
3. No intermediate storage NIS: There is no storage capacity available.

Depending on whether for a given intermediate product, the stock-keeping facility is a single-product or a multi-product storage, we distinguish between two *storage homogeneity settings*:

1. Dedicated intermediate storage DIS: The product is stocked in a homogenous storage.
2. Shared intermediate storage SIS: The product is stocked together with other products in a heterogenous storage.

For what follows, we assume that the storage facilities are replenished and depleted according to a first-in first-out strategy. The *storage time settings* for a given intermediate product can then be classified according to:

1. Unlimited wait UW: The product can be consumed immediately after production and can be stored for an unlimited time.
2. Finite wait FW: A quarantine time $q \geq 0$ says that any unit of the product can be consumed q units of time after its production at the earliest. Symmetrically, a shelf life time $s \geq 0$ implies that any unit of the product must be consumed s units of time after its production at the latest.
3. Zero wait ZW: Any unit of the product has to be consumed immediately after production. In case of continuous production mode, this means that the total production and the total consumption rates for the product must coincide at any point in time.

To the best of our knowledge, the above storage settings have not been treated in a unifying framework thus far. In Section 2 we present a basic model for scheduling continuous material flows. In Section 3 we explain how to model the different storage capacity, storage homogeneity, and storage time settings, respectively. Furthermore, we show that batch production mode and the use of renewable resources like manpower or processing units are contained as special cases.

2 MODEL

Let $\mathcal{O}$ be the set of operations to be scheduled. The duration of operation $i \in \mathcal{O}$ is denoted by p_i and the start time by S_i. The vector $S = (S_i)_{i \in \mathcal{O}}$ is called (production) schedule. During its execution in time interval $[S_i, S_i + p_i[$, operation i consumes input products and produces output products at constant rates. $\mathcal{C}$ is the set of cumulative resources (cf. Neumann and Schwindt 1999 and Schwindt and Trautmann 2000) stocking the intermediates produced and consumed by the operations $i \in \mathcal{O}$. ρ_{ik} denotes the—possibly negative—increase in the inventory level of resource k after the end of operation i and will be referred to as the demand of operation i for resource k. If $\rho_{ik} > 0$, operation i replenishes resource k, and if $\rho_{ik} < 0$, operation i depletes resource k. For each cumulative resource, there is a prescribed minimum inventory level $\underline{R}_k$ and a prescribed maximum inventory level $\overline{R}_k \geq \underline{R}_k$. By

$$x_i(S,t) = \begin{cases} 0, \text{ if } t < S_i \\ 1, \text{ if } t \geq S_i + p_i \\ (t - S_i)/p_i, \text{ otherwise} \end{cases}$$

we denote the portion of operation $i \in \mathcal{O}$ that has been processed by time t. For a given schedule S, the inventory level in resource $k \in \mathcal{C}$ at time $t \geq 0$ is

$$\rho_k(S,t) := \sum_{i \in \mathcal{O}} \rho_{ik} x_i(S,t)$$

In addition, we have a set $E \subset \mathcal{O} \times \mathcal{O}$ of operation pairs (i,j) with associated weights δ_{ij}. A nonnegative weight δ_{ij} corresponds to a prescribed minimum time lag of $d_{ij}^{min} = \delta_{ij}$ units of time between the starts of operations i and j. A negative weight δ_{ij} implies a maximum time lag of $d_{ji}^{max} = -\delta_{ij}$ units of time between the starts of operations j and i. The temporal constraints given by minimum and maximum time lags can be written as

$$S_j - S_i \geq \delta_{ij} \quad ((i,j) \in E)$$

With $f(S)$ denoting the objective function, the production scheduling problem can be formulated as follows:

$$\begin{array}{lll} \text{Minimize} & f(S) & \\ \text{subject to} & \underline{R}_k \leq \rho_k(S,t) \leq \overline{R}_k & (k \in \mathcal{C},\ t \geq 0) \quad (1) \\ & S_j - S_i \geq \delta_{ij} & ((i,j) \in E) \quad (2) \\ & S_i \geq 0 & (i \in \mathcal{O}) \quad (3) \end{array}$$

A solution procedure of the branch-and-bound type for this problem is discussed in Schwindt and Trautmann (2002).

3 APPLICATIONS

In Subsections 3.1, 3.2, and 3.3 we show how to formulate the different storage capacity, storage homogeneity, and storage time settings, respectively, from

Section 1 on the basis of model presented in Section 2. Subsection 3.4 is concerned with discrete resources like storage facilities in batch production mode, manpower, and processing units.

3.1 Storage Capacity Settings

1. UIS. Each intermediate product corresponds to one cumulative resource $k \in \mathcal{C}$ with infinite maximum inventory level $\overline{R}_k$ and the minimum inventory level $\underline{R}_k$ being equal to the safety stock of the product. The demand ρ_{ik} of an operation i for resource k equals the (negative) amount of the intermediate produced (consumed).

2. FIS. The case of finite intermediate storage differs from the UIS case in that the maximum inventory level $\overline{R}_k$ of the corresponding cumulative resource k coincides with the capacity of the respective storage facility.

3. NIS. The case of no intermediate storage corresponds to the FIS case with $\underline{R}_k = \overline{R}_k = 0$.

3.2 Storage Homogeneity Settings

1. DIS. Each cumulative resource k corresponds to one intermediate product to be stocked. The storage capacity $\overline{R}_k$ coincides with the maximum inventory of the product.

2. SIS. We consider the case of a heterogeneous intermediate storage facility keeping $\pi > 1$ different products. This setting is modelled by $\pi + 1$ cumulative resources. The resources $k = 1, \ldots, \pi$ correspond to one intermediate product each. The minimum inventory levels $\underline{R}_k$ of resources $k = 1, \ldots, \pi$ equal the safety stocks for the respective products. The maximum inventory levels $\overline{R}_k$ of those resources k are infinite. The capacity of the storage facility is taken into account by the maximum inventory level $\overline{R}_{\pi+1}$ of resource $\pi + 1$. The minimum inventory level $\underline{R}_{\pi+1}$ equals 0. The production of an intermediate replenishes the respective resource k as well as resource $\pi + 1$. Analogously, the consumption of an intermediate depletes the respective resource k as well as resource $\pi + 1$.

3.3 Storage Time Settings

1. UW. If an intermediate product can be consumed immediately after production and can be stored for an unlimited waiting time, no additional constraints are needed.

2. FW. We first consider the case of an intermediate product with a quarantine time of $q \geq 0$ units of time. This can be modelled using two cumulative resources k and k' (cf. left-hand part of Fig. 1). Resource k represents the storage facility, and thus the minimum and maximum inventory levels $\underline{R}_k$ and $\overline{R}_k$ are chosen according to the storage capacity settings for the product. For resource k', we set zero minimum and infinite maximum inventory level, i.e., $\underline{R}_{k'} = 0$ and $\overline{R}_{k'} = \infty$. For each operation i producing the intermediate,

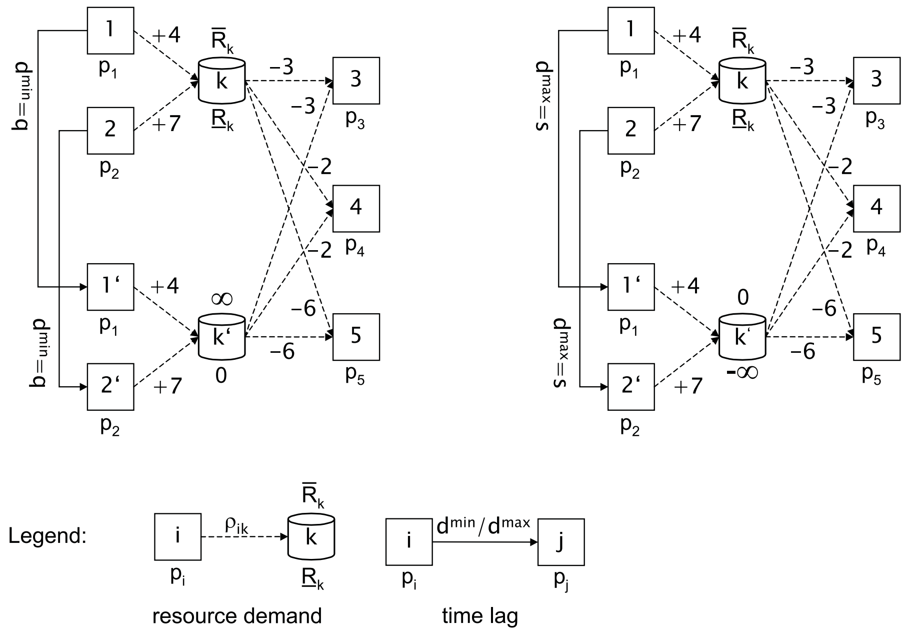

Figure 1 – Quarantine and shelf life times

we introduce a fictitious operation i' with duration $p_{i'} = p_i$. Operations i and i' are linked by a minimum time lag of q units of time, i.e., $d_{ii'}^{min} = q$. The demand of fictitious operation i' for resource k' equals the replenishment of resource k by operation i, i.e., $\rho_{i'k'} = \rho_{ik}$. Each consuming operation j depletes resources k and k' by the same amount, i.e., $\rho_{jk'} = \rho_{jk}$. Since the replenishment of resource k' occurs q units of time after the replenishment of resource k at the earliest, any consuming operation j, which depletes both k and k', cannot be started earlier than q units of time after the production of the intermediate (otherwise, the inventory in resource k' would fall below the minimum inventory level of 0). Thus, there is a minimum time lag of q units of time between the production and the consumption of the product.

Next, we deal with the case of an intermediate product with a shelf life time of $s \geq 0$ units of time. Analogously to the case of a quarantine time, we introduce two cumulative resources k and k' (cf. right-hand part of Fig. 1). The minimum inventory level of resource k' is now $\underline{R}_{k'} = -\infty$, and the maximum inventory level is $\overline{R}_{k'} = 0$. For each operation i producing the intermediate, a fictitious operation i' with duration $p_{i'} = p_i$ and resource demand $\rho_{i'k'} = \rho_{ik}$ is introduced. Operations i and i' are now linked by a maximum time lag of s units of time, i.e., $d_{ii'}^{max} = s$. Again, each consuming operation j depletes resources k and k' by the same amount, i.e., $\rho_{jk'} = \rho_{jk}$. Since the replenishment of resource k' occurs s units of time after the replenishment of resource k at the latest, any consuming operation j must be started no later than s units of time after the production of the intermediate (otherwise, the inventory in resource k' would exceed the maximum inventory level of 0). Thus, there is a maximum time lag of s units of time between the production and the consumption of the product. We notice that $\sum_{i \in \mathcal{O}} \rho_{ik} = 0$ is a necessary condition on the existence of a feasible schedule S. In particular, this means that $\underline{R}_k = 0$.

3. ZW. The zero wait setting corresponds to both a quarantine and a shelf life time of $q = s = 0$, which can be modelled as described above. Note that instead of defining an additional resource k' it is sufficient to set $\overline{R}_k := \underline{R}_k := 0$. Hence, the ZW and the NIS settings are equivalent.

3.4 Discrete resources

Storage facilities operated in batch production mode can be modelled using the model of Section 2 as follows. We replace each operation $i \in \mathcal{O}$ by two dummy operations i' and i'' of duration 0. i' represents the start and i'' the completion of operation i. Duration p_i is modelled by a minimum and a maximum time lag $d^{min}_{i'i''} = d^{max}_{i'i''} = p_i$ saying that the completion of i occurs precisely p_i units of time after its start. Recall that in batch production mode, the input of an operation is consumed at its start and the output arises at its completion. Thus, if $\rho_{ik} < 0$, we have $\rho_{i'k} = \rho_{ik}$, $\rho_{i''k} = 0$ and if $\rho_{ik} > 0$, we have $\rho_{i'k} = 0$, $\rho_{i''k} = \rho_{ik}$. Storage capacity, storage homogeneity, and storage time settings can be modelled as described above.

Workers of the same qualification and processing units of the same type are grouped to a pool. A pool corresponds to a (renewable) resource whose availability is independent of its previous utilization. Each pool is modelled as a cumulative resource k with a maximum inventory level $\overline{R}_k$ equal to the number of workers or processing units in the pool. The minimum inventory level is $\underline{R}_k = 0$. Again, each operation i is replaced by two dummy operations i' and i'' representing the start and the completion of operation i. The replenishment of k by operation i' and the depletion of k by operation i'' coincide with the number of workers required.

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