A Closed-Loop Approach to Continuous Process Scheduling

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Outline

1. Process scheduling problem
2. Decomposition into planning and scheduling
   - Operations planning problem
   - Operations scheduling problem
3. Closed-loop approach
   - Basic idea
   - Operations re-planning model
   - Performance analysis
4. Conclusions

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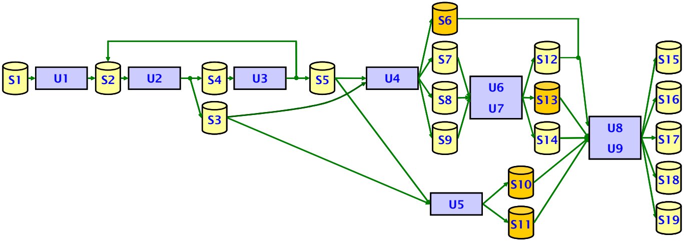
Problem definition I

Equipment

- Multistage continuous multiproduct production plant
- Multipurpose processing units $u \in \mathcal{U}$ operated in continuous mode
- Dedicated storage facilities $s \in \mathcal{S}$ of limited capacity



Problem definition II

Operations and states

- Final products arise from sequences of operations executed on dedicated processing units
- During execution of operation materials continuously flow through processing unit
- Each operation $i \in \mathcal{O}$ transforms input states $s \in \mathcal{S}^{i-}$ into output states $s \in \mathcal{S}^{i+}$
- Sequence-dependent cleaning times ϑ_{ij} on processing units
- Processing times π_i, production rates γ_i, input and output proportions α_{is} (operating conditions), and start times σ_i subject to decision

Problem definition III

Continuous process scheduling problem

Determine production schedule

- operating conditions of operations
- start times of operations

such that

- prescribed bounds for operating conditions are observed
- given primary requirements for final products are satisfied
- no processing unit processes more than one operation at a time
- processing units can be cleaned between consecutive operations
- sufficient amount of input states and sufficient storage space for output states are available during the execution of each operation
- schedule length does not exceed planning horizon
- objective function is optimized (makespan, tardiness, profit)

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Decomposition into planning and scheduling

Operations planning problem

Determine operating conditions of operations subject to

- bounds for operating conditions
- constraints on final inventory levels
- constraints anticipating storage-capacity restrictions

⇓

Operations scheduling problem

Determine start times of operations subject to

- limited availability of processing units, input states, and storage space for output products
- sequence-dependent cleaning times
- upper bound on schedule length

Basic NLP planning model

Model (OP)

$$
\begin{aligned}
\text{Min. } & f^p(\alpha, \gamma, \pi) \\
\text{s.t. } & \sum_{s \in \mathcal{S}^{i+}} \alpha_{is} = - \sum_{s \in \mathcal{S}^{i-}} \alpha_{is} = 1 && (i \in \mathcal{O}) \\
& \underline{\alpha}_{is} \leq \alpha_{is} \leq \overline{\alpha}_{is} && (i \in \mathcal{O}; s \in \mathcal{S}^i) \\
& \underline{\gamma}_i \leq \gamma_i \leq \overline{\gamma}_i && (i \in \mathcal{O}) \\
& \underline{\pi}_i \leq \pi_i \leq \overline{\pi}_i && (i \in \mathcal{O}) \\
& \delta_s \leq \rho_s^0 + \sum_{i \in \mathcal{O}^s} \alpha_{is}\gamma_i\pi_i \leq \overline{\rho}_s && (s \in \mathcal{S}) \\
& \alpha_{is}\gamma_i = -\alpha_{js}\gamma_j && (s \in \mathcal{S}; i \in \mathcal{O}^{s+}; j \in \mathcal{O}^{s-}) \\
& \alpha_{is}\gamma_i \max_{j,k \in \mathcal{O}^{s-}} \vartheta_{jk} \leq \overline{\rho}_s && (s \in \mathcal{S} : \overline{\rho}_s > 0; i \in \mathcal{O}^{s+})
\end{aligned}
$$

Basic NLP planning model

Model (*OP*)

$$\text{Min.} \quad \tilde{f}^p(\alpha,\gamma,\pi,\zeta) = \|\zeta^1 + \zeta^2\|_1 + \varepsilon f^p(\alpha,\gamma,\pi)$$

$$\text{s.t.} \quad \sum_{s\in\mathcal{S}^{i+}} \alpha_{is} = -\sum_{s\in\mathcal{S}^{i-}} \alpha_{is} = 1 \qquad (i \in \mathcal{O})$$

$$\underline{\alpha}_{is} \le \alpha_{is} \le \overline{\alpha}_{is} \qquad (i \in \mathcal{O}; s \in \mathcal{S}^i)$$

$$\underline{\gamma}_i \le \gamma_i \le \overline{\gamma}_i \qquad (i \in \mathcal{O})$$

$$\underline{\pi}_i \le \pi_i \le \overline{\pi}_i \qquad (i \in \mathcal{O})$$

$$\delta_s \le \rho_s^0 + \sum_{i\in\mathcal{O}^s} \alpha_{is}\gamma_i\pi_i \le \overline{\rho}_s \qquad (s \in \mathcal{S})$$

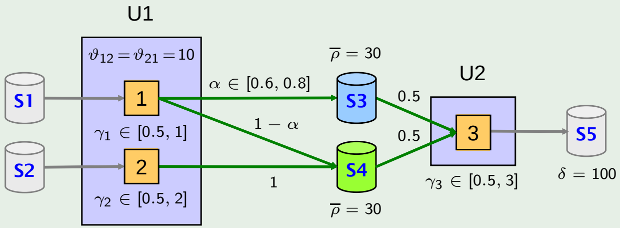
$$\alpha_{is}\gamma_i = -\alpha_{js}\gamma_j + \zeta_{ijs}^1 - \zeta_{ijs}^2 \qquad (s \in \mathcal{S}; i \in \mathcal{O}^{s+}; j \in \mathcal{O}^{s-})$$

$$\alpha_{is}\gamma_i \max_{j,k\in\mathcal{O}^{s-}} \vartheta_{jk} \le \overline{\rho}_s \qquad (s \in \mathcal{S} : \overline{\rho}_s > 0; i \in \mathcal{O}^{s+})$$

$$\zeta_{ijs}^1, \zeta_{ijs}^2 \ge 0 \qquad (s \in \mathcal{S}; i \in \mathcal{O}^{s+}; j \in \mathcal{O}^{s-})$$

Example

Example (Make-and-mix plant)



Operating conditions for $f^p(\alpha, \gamma, \pi) = \pi_1 + \pi_2 + \pi_3$

$i = 1$: $\gamma_1 = 0.83$ $\pi_1 = 100.0$ $\alpha = 0.6$
$i = 2$: $\gamma_2 = 0.5$ $\pi_2 = 33.3$
$i = 3$: $\gamma_3 = 1.0$ $\pi_3 = 100.0$

Operations scheduling problem I

Solution procedures

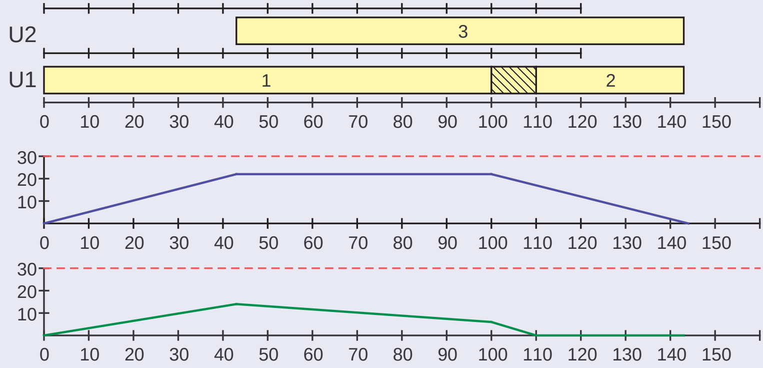
- Branch-and-bound method: Neumann K, S., Trautmann N (2005)
- Priority-rule based method: Herrmann S, S. (2007)
- Exact MILP model ($OS(\alpha, \gamma, \pi)$): Herrmann S, S. (2008)

Proposition (Feasibility of scheduling problem)

If all material flows are acyclic and $\zeta^1 + \zeta^2 = 0$, then there exists a feasible solution to the operations scheduling problem, which can be obtained in polynomial time.

Operations scheduling problem II

Example: optimal schedule for $f(\alpha, \gamma, \pi, \sigma) = \max_{i \in \mathcal{O}}(\sigma_i + \pi_i) = C_{\max}$



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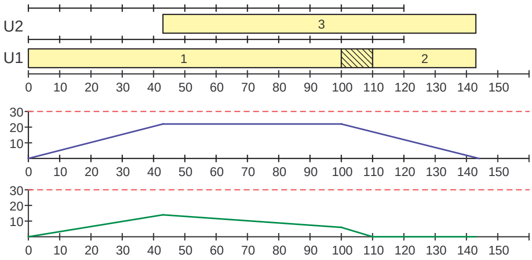
Basic idea I

Motivation

- Maximum inventory levels depend on operations sequence
- To facilitate generation of feasible schedule, planning model (OP) aligns rates $|\alpha_{is}\gamma_i|$ of producing and consuming operations
- Production rates generally unnecessarily small



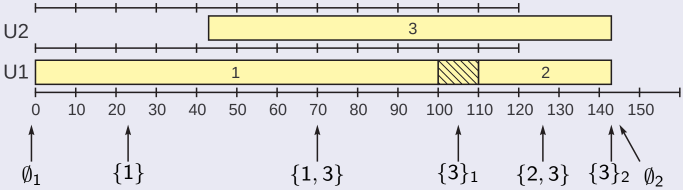
Basic idea II

Basic idea

- Return to planning phase after scheduling
- Fix sequence of start and completion events and replace alignment of rates by exact material-availability and storage-capacity constraints in planning problem
- Inventory levels at support points can be expressed as sums of amounts produced and consumed by active sets (antichains)
- Associate decision variable $\pi_{\mathcal{A}}$ with each active set $\mathcal{A}$ providing its duration
- Processing times π_i and start times σ_i uniquely given by sequence and durations of active sets

Active sets I

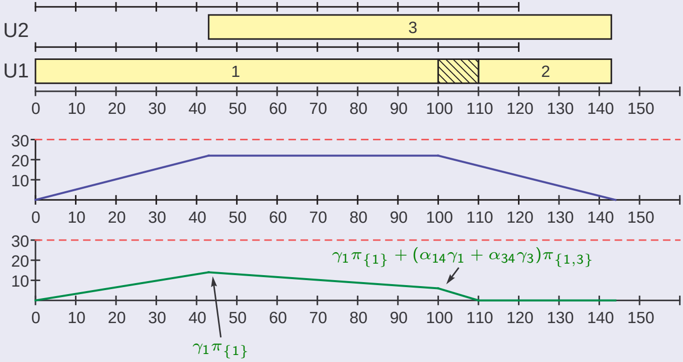
Example (cont'd)



- $\mathcal{B} = \{\mathcal{A}_1, \ldots, \mathcal{A}_\nu\} = \{\emptyset_1, \{1\}, \{1,3\}, \{3\}_1, \{2,3\}, \{3\}_2, \emptyset_2\}$
- $\pi_1 = \pi_{\{1\}} + \pi_{\{1,3\}}$, $\pi_2 = \pi_{\{2,3\}}$, $\pi_3 = \pi_{\{1,3\}} + \pi_{\{3\}_1} + \pi_{\{2,3\}} + \pi_{\{3\}_2}$
- $\sigma_1 = \pi_{\emptyset_1}$, $\sigma_2 = \pi_{\emptyset_1} + \pi_{\{1\}} + \pi_{\{1,3\}} + \pi_{\{3\}_1}$, $\sigma_3 = \pi_{\emptyset_1} + \pi_{\{1\}}$

Active sets II

Example (cont'd)



Operations re-planning model

Model ($ORP(\sigma')$)

$$
\begin{array}{lll}
\text{Min.} & f(\alpha,\gamma,\pi,\sigma) & \\
\text{s.t.} & \sum_{s\in\mathcal{S}^{i+}} \alpha_{is} = -\sum_{s\in\mathcal{S}^{i-}} \alpha_{is} = 1 & (i\in\mathcal{O}) \\
& \underline{\alpha}_{is} \leq \alpha_{is} \leq \overline{\alpha}_{is} & (i\in\mathcal{O}; s\in\mathcal{S}^i) \\
& \underline{\gamma}_i \leq \gamma_i \leq \overline{\gamma}_i & (i\in\mathcal{O}) \\
& \underline{\pi}_i \leq \pi_i \leq \overline{\pi}_i & (i\in\mathcal{O}) \\
& \delta_s \leq \rho_s^0 + \sum_{i\in\mathcal{O}^s} \alpha_{is}\gamma_i\pi_i \leq \overline{\rho}_s & (s\in\mathcal{S}) \\
& \pi_i = \sum_{\mathcal{A}\in\mathcal{B}^s(\nu_s):i\in\mathcal{A}} \pi_{\mathcal{A}} & (i\in\mathcal{O}; s\in\mathcal{S}^i) \\
& \sigma_i = \sum_{\mathcal{A}\in\mathcal{B}^s(\mu)} \pi_{\mathcal{A}} & (s\in\mathcal{S}; \mu=1,\ldots,\nu^s-1; i\in\mathcal{A}^s_{\mu+1}\setminus\mathcal{A}^s_{\mu}) \\
& 0 \leq \rho_s^0 + \sum_{\mathcal{A}\in\mathcal{B}^s(\mu)} \sum_{i\in\mathcal{A}} \alpha_{is}\gamma_i\pi_{\mathcal{A}} \leq \overline{\rho}_s & (s\in\mathcal{S}; \mu=1,\ldots,\nu^s-1) \\
& \sigma_j - \sigma_i \geq \pi_i + \vartheta_{ij} & (u\in\mathcal{U};\ i,j\in\mathcal{O}^u : \sigma'_j \geq \sigma'_i) \\
& \sigma_i + \pi_i \leq \tau & (i\in\mathcal{O}) \\
& \pi_{\mathcal{A}} \geq 0 & (s\in\mathcal{S}; \mathcal{A}\in\mathcal{B}^s(\nu_s))
\end{array}
$$

Closed-loop method I

Closed-loop method

Input: process scheduling problem
Output: feasible production schedule $(\alpha, \gamma, \pi, \sigma')$

determine initial operating conditions (α, γ, π) by solving basic planning model (OP);
repeat
compute schedule σ' by solving resulting scheduling problem ($OS(\alpha, \gamma, \pi)$);
re-optimize operating conditions (α, γ, π) with operations re-planning planning model ($ORP(\sigma')$);
until fixed point $(\alpha, \gamma, \pi, \sigma')$ has been reached;
return feasible production schedule $(\alpha, \gamma, \pi, \sigma')$;

Closed-loop method II

Proposition (Monotonicity and finiteness)

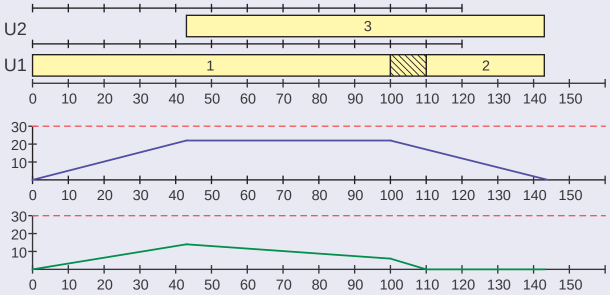
Provided that problems (OP) and (OS(α, γ, π)) are feasible, the sequence of generated objective function values $f(\alpha, \gamma, \pi, \sigma')$ is nonincreasing. The closed-loop method attains a fixed point after a finite number of iterations.

Proof.

The monotonicity follows from the feasibility of the preceding production schedule $(\alpha, \gamma, \pi, \sigma')$ with respect to (ORP(σ')). The feasible region of (ORP(σ')) only depends on the sequence of active sets $\mathcal{A}_\mu$ induced by σ'. In conjunction with the monotonicity this provides the finiteness. □

Example (cont'd)

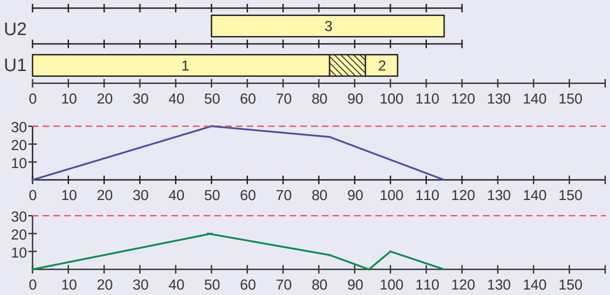
Initial schedule after scheduling



$\gamma_1 = 0.83$
$\alpha = 0.6$
$\gamma_2 = 0.5$
$\gamma_3 = 1.0$
$C_{\max} = 143.3$

Example (cont'd)

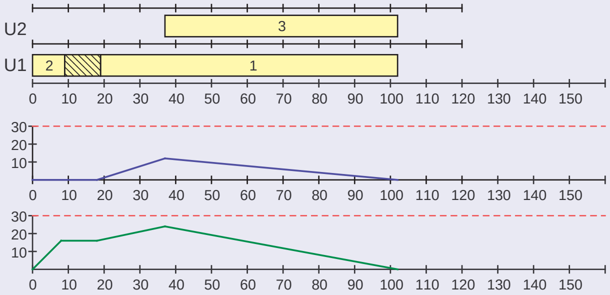
Second schedule after re-planning



$\gamma_1 = 1.0$
$\alpha = 0.6$
$\gamma_2 = 2.0$
$\gamma_3 = 1.54$
$C_{\max} = 115.0$

Example (cont'd)

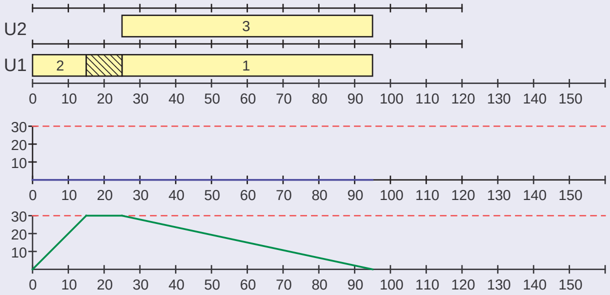
Third schedule after scheduling



$\gamma_1 = 1.0$
$\alpha = 0.6$
$\gamma_2 = 2.0$
$\gamma_3 = 1.54$
$C_{max} = 101.7$

Example (cont'd)

Fourth and final schedule after re-planning / scheduling

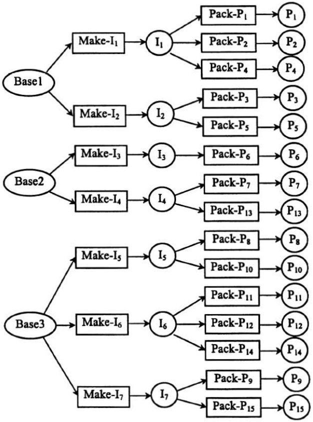


$\gamma_1 = 1.0$
$\alpha = 0.71$
$\gamma_2 = 2.0$
$\gamma_3 = 1.43$
$C_{\max} = 95.0$

Performance analysis I

Test bed: FMCG case study

- Case study from FMCG industry (Méndez and Cerdá 2002)
- Objective makespan minimization
- 10 instances with varying primary requirements for final products
- Planning models solved under GAMS using NLP solver CONOPT3
- Scheduling model solved under GAMS using MILP solver Cplex 11.0
- Lower bounds computed with (relaxation of) MILP model by Méndez and Cerdá 2002, time limit 3600.0 sec
- Pentium IV PC, 3.8 GHz, 2 GB RAM



Performance analysis II

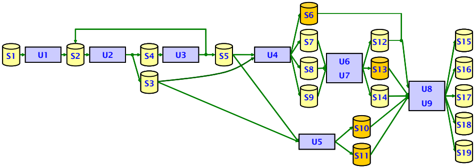
Computational results: FMCG case study

d_1–d_5	d_6–d_{10}	d_{11}–d_{15}	C^{ini}_{max}	C^{fin}_{max}	n_{it}	t_{cpu} [sec]	C^{milp}_{max}
Original demands			224.9	73.8	4	60.8	71.3
50	50	50	112.8	77.9	5	126.5	77.8
100	100	100	213.7	153.3	6	166.0	151.7
150	150	150	367.2	229.5	5	199.9	225.9
50	100	150	277.7	225.9	4	112.1	225.9
50	150	100	244.6	153.3	6	227.0	151.7
100	50	150	258.2	225.9	4	71.2	225.9
100	150	50	207.1	188.6	4	108.6	188.6
150	50	100	276.9	151.7	6	176.1	151.7
150	100	50	276.9	192.6	4	46.4	188.6

Performance analysis III

Test bed: Kallrath case study

- Case study of Kallrath 2002 adapted to continuous production mode
- Objective makespan minimization
- 8 instances with varying primary requirements for final products
- Planning models solved under GAMS using NLP solver CONOPT3
- Scheduling model solved under GAMS using MILP solver Cplex 11.0
- Lower bounds computed with (tightened version of) MILP model by Giannelos and Georgiadis 2002, time limit 3600.0 sec
- Pentium IV PC, 3.8 GHz, 2 GB RAM



Performance analysis IV

Computational results: Kallrath case study

d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	C_{max}^{ini}	C_{max}^{fin}	n_{it}	t_{cpu} [sec]	C_{max}^{milp}
15	15	15	15	15	31.7	26.7	4	29.7	26.8
20	20	20	20	20	42.3	32.7	6	98.6	37.6
25	25	25	25	25	52.9	42.5	4	37.5	59.6
30	30	20	20	40	65.2	47.7	6	73.8	52.2
30	30	20	30	30	65.2	47.7	7	81.6	48.1
30	40	20	40	30	78.7	55.2	4	34.5	57.8
40	20	20	20	40	56.8	47.0	4	33.6	46.9
40	30	20	30	40	72.5	54.1	5	47.9	46.9

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Summary

- Decomposition approach for process scheduling of continuous multiproduct plants
- Planning problem: determine operating conditions of operations
- Scheduling problem: schedule operations on processing units
- Closed-loop method: re-optimize operating conditions subject to constraints on active sets
- Fixed point reached in finite number of iterations
- Good schedules within reasonable amount of time, high reliability

Future research

- Tests for alternative objective functions (revenues, profit, tardiness)
- Metaheuristic search procedure performing perturbation steps after convergence

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Backup: The scheduling model

Model $(OS(\alpha, \gamma, \pi))$

$$
\begin{array}{lll}
\text{Min.} & f(\alpha,\gamma,\pi,\sigma) & \\
\text{s.t.} & 0 \le \sigma_i \le \tau - \pi_i & (i \in \mathcal{O}) \\
& \pi_i + \vartheta_{ij} - \tau(1 - z_{ij}) \le \sigma_j - \sigma_i \le -\pi_j - \vartheta_{ji} + \tau z_{ij} & (u \in \mathcal{U}; i,j \in \mathcal{O}^u : i < j) \\
& 0 \le x_{ijs} \le 1 & (s \in \mathcal{S}; i,j \in \mathcal{O}^s) \\
& x_{ijs} \ge y_{ijs} & (s \in \mathcal{S}; i \in \mathcal{O}^{s-}; j \in \mathcal{O}^s) \\
& x_{ijs} \le 1 - y_{ijs} & (s \in \mathcal{S}; i \in \mathcal{O}^{s+}; j \in \mathcal{O}^s) \\
& \pi_i - \tau(1 - y_{ijs}) \le \sigma_j + \pi_j - \sigma_i \le \pi_i x_{ijs} + \tau y_{ijs} & (s \in \mathcal{S}; i \in \mathcal{O}^{s-}; j \in \mathcal{O}^{s-}) \\
& \pi_i - \tau(1 - y_{ijs}) \le \sigma_j - \sigma_i \le \pi_i x_{ijs} + \tau y_{ijs} & (s \in \mathcal{S}; i \in \mathcal{O}^{s-}; j \in \mathcal{O}^{s+}) \\
& \pi_i x_{ijs} - \tau y_{ijs} \le \sigma_j + \pi_j - \sigma_i \le \tau(1 - y_{ijs}) & (s \in \mathcal{S}; i \in \mathcal{O}^{s+}; j \in \mathcal{O}^{s-}) \\
& \pi_i x_{ijs} - \tau y_{ijs} \le \sigma_j - \sigma_i \le \tau(1 - y_{ijs}) & (s \in \mathcal{S}; i \in \mathcal{O}^{s+}; j \in \mathcal{O}^{s+}) \\
& \rho_s^0 + \sum_{i \in \mathcal{O}^s} \alpha_{is}\gamma_i\pi_i x_{ijs} \ge 0 & (s \in \mathcal{S}; j \in \mathcal{O}^s) \\
& y_{ijs} \in \{0,1\} & (s \in \mathcal{S}; i,j \in \mathcal{O}^s) \\
& z_{ij} \in \{0,1\} & (u \in \mathcal{U}; i,j \in \mathcal{O}^u : i < j)
\end{array}
$$