Large-Scale Short-Term Planning in Chemical Batch Production

N. Trautmann, University of Bern
Ch. Schwindt, Clausthal University of Technology

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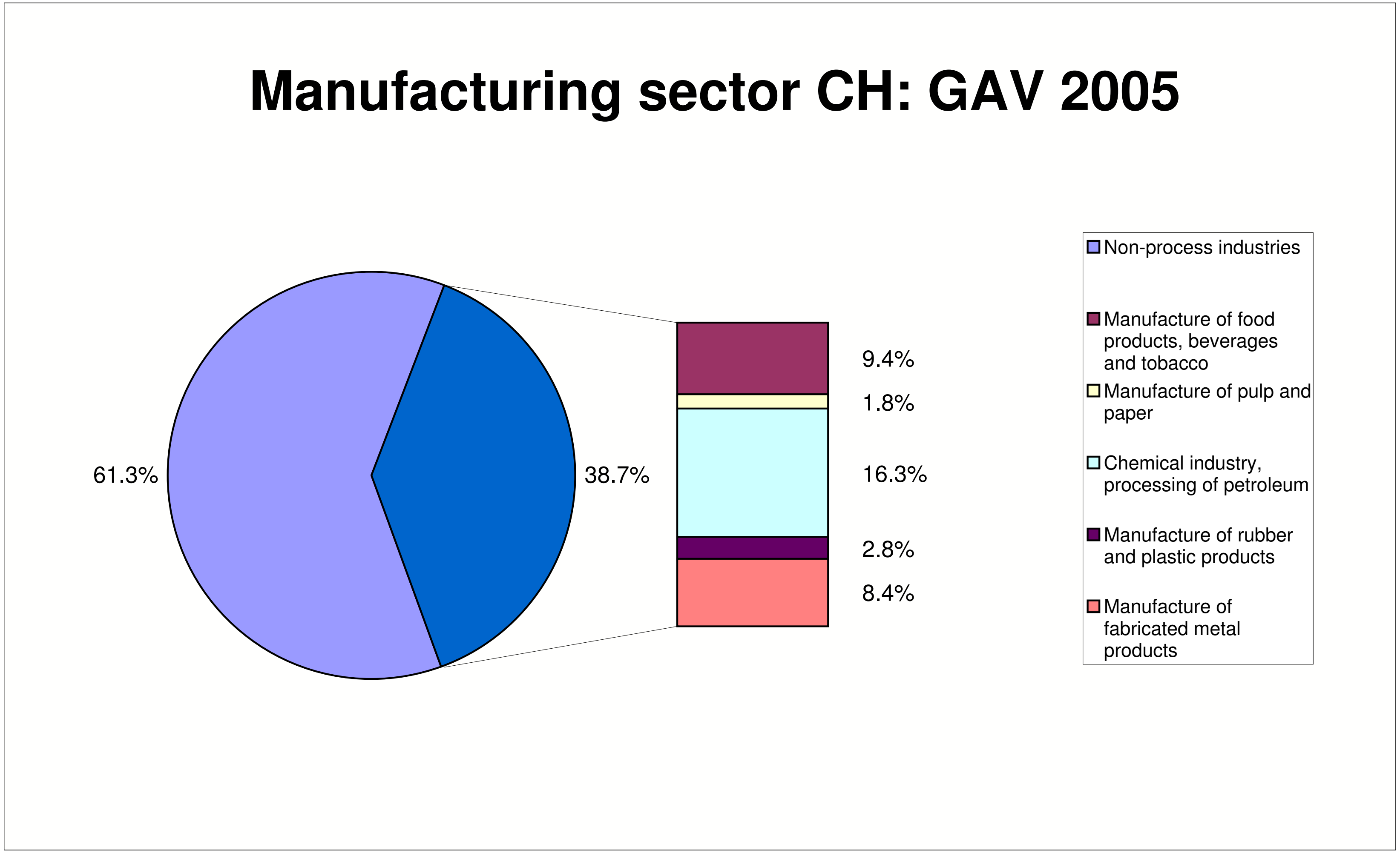
Outline

1. Short-term planning problem
2. Overview of cyclic approach
3. Cyclic batching
4. Cyclic batch scheduling and concatenation
5. Performance analysis
6. Conclusions

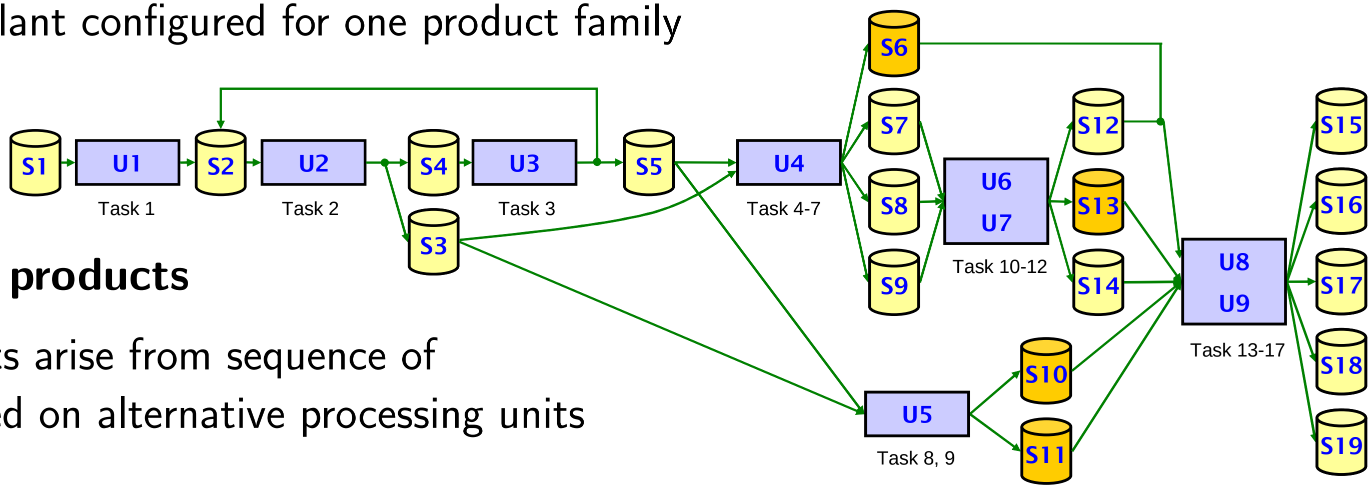


Source: Swiss Federal Statistical Office

1 Short-term planning problem

Equipment

- Multipurpose processing units operated in batch mode
- Dedicated storage facilities of given capacity
- Production plant configured for one product family



Processes and products

- Final products arise from sequence of tasks executed on alternative processing units
- Each task
  - ▷ consumes and produces one or several products
  - ▷ may be executed several times
- Batch sizes as well as input and output proportions subject to decision
- Intermediates may be chemically instable (perishable products)
- Sequence-dependent cleaning times

Short-term planning problem

Determine

- batch size, input/output proportions, and number of executions for each task (operations)
- assignment of operations to processing units
- start times of operations

such that

- given primary requirements for final products are satisfied
- prescribed intervals for batch sizes and input/output proportions are observed
- no processing unit processes more than one operation at a time
- processing units are cleaned between consecutive operations
- sufficient amount of each input product is available at the start of each operation
- sufficient storage space for output products is available at the completion of each operation
- all perishable products are consumed immediately after production
- makespan is minimized

Related literature

Surveys on short-term planning in the process industries

- Kallrath J (2002) Planning and scheduling in the process industry. OR Spectrum 24:219–250
- Burkard RE, Hatzl J (2006) Review, extensions and computational comparison of MILP formulations for scheduling of batch processes. Computers and Chemical Engineering 29:1752–1769

Short-term planning of batch plants: Monolithic approaches

- Kondili E, Pantelides CC, Sargent RWH (1993) A general algorithm for short-term scheduling of batch operations: 1. MILP Formulation. Computers and Chemical Engineering 17:211–227
- Ierapetritou MG, Floudas CA (1998) Effective continuous-time formulation for short-term scheduling: 1. Multipurpose batch processes. Industrial & Engineering Chemistry Research 37:4341–4359
- Blömer F, Günther H-O (2000) LP-based heuristics for scheduling chemical batch process. International Journal of Production Research 38:1029–1051

Short-term planning of batch plants: Decomposition approaches

- Brucker P, Hurink J (2000) Solving a chemical batch scheduling problem by local search. Annals of Operations Research 96:17–36
- Neumann K, Schwindt C, Trautmann N (2002) Advanced production scheduling for batch plants in process industries. OR Spectrum 24:251–279
- Schwindt C, Trautmann N (2004) A priority-rule based method for batch production scheduling in the process industries. In: Ahr D, Fahrion R, Oswald M, Reinelt G (eds) Operations Research Proceedings 2003. Springer, Berlin, pp. 111–118
- Gentner K, Neumann K, Schwindt C, Trautmann N (2004) Batch production scheduling in the process industries. In: Leung JYT (ed) Handbook of Scheduling: Algorithms, Models and Performance Analysis. CRC Press, Boca Raton, pp. 48.1–48.21

2 Overview of cyclic approach

Cyclic batching

- **Given**:
  - ▷ Primary requirements, initial inventory levels
  - ▷ Bounds on batch sizes
  - ▷ Bounds on input/output proportions
  - ▷ Upper bound on total number of task executions
- **Sought**:
  - ▷ Operations of one cycle
  - ▷ Number of cycles
- **Solution method**:
  - ▷ Formulation as MINLP of moderate size
  - ▷ Standard software

Cyclic batch scheduling

- **Given**:
  - ▷ Processing units
  - ▷ Storage facilities
  - ▷ Cleaning times
  - ▷ Operations of one cycle
- **Sought**:
  - ▷ Assignment of operations to processing units
  - ▷ Start times of operations in cyclic subschedule
- **Solution method**:
  - ▷ Formulation as resource-constrained scheduling problem
  - ▷ Branch-and-bound or priority-rule based method

Concatenation

- **Given**:
  - ▷ Cyclic subschedule
  - ▷ Number of cycles
- **Sought**:
  - ▷ Start times of operations in complete schedule
- **Solution method**:
  - ▷ Formulation as temporal scheduling problem
  - ▷ Standard network flow algorithm (longest path calculations)

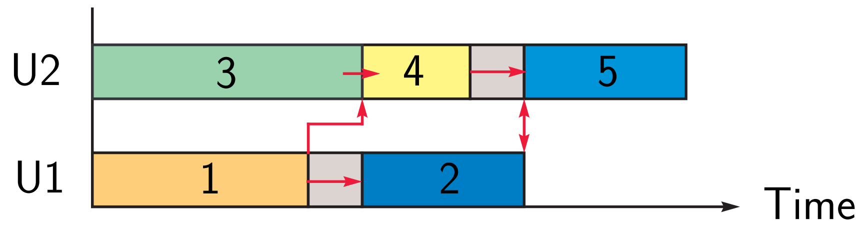
3 Cyclic batching

Mixed-integer nonlinear program

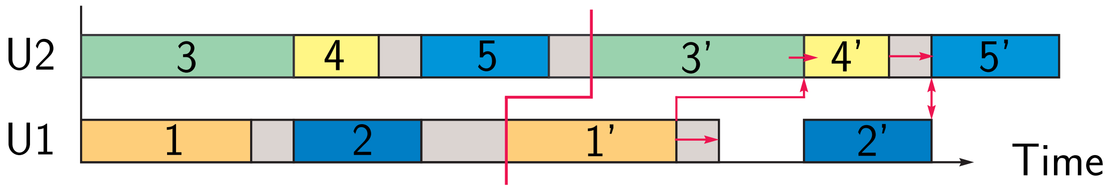
$$
\begin{cases}
\text{Minimize} & \xi \sum_{\tau\in\mathcal{T}} p_\tau \varepsilon_\tau & & \\
\text{subject to} & \underline{\alpha}_{\tau\pi} \le \alpha_{\tau\pi} \le \overline{\alpha}_{\tau\pi} & (\tau\in\mathcal{T},\ \pi\in\mathcal{P}_\tau^- \cup \mathcal{P}_\tau^+) & (1)\\
& \underline{\beta}_\tau \le \beta_\tau \le \overline{\beta}_\tau & (\tau\in\mathcal{T}) & (2)\\
& \sum_{\pi\in\mathcal{P}_\tau^+} \alpha_{\tau\pi} = -\sum_{\pi\in\mathcal{P}_\tau^-} \alpha_{\tau\pi} = 1 & (\tau\in\mathcal{T}) & (3)\\
& \sum_{\tau\in\mathcal{T}_\pi^-\cup\mathcal{T}_\pi^+} \alpha_{\tau\pi}\beta_\tau\varepsilon_\tau = 0 & (\pi\in\mathcal{P}^i) & (4)\\
& \xi \sum_{\tau\in\mathcal{T}_\pi^-\cup\mathcal{T}_\pi^+} \alpha_{\tau\pi}\beta_\tau\varepsilon_\tau \ge \rho_\pi & (\pi\in\mathcal{P}\setminus\mathcal{P}^i) & (5)\\
& \alpha_{\tau\pi}\beta_\tau = -\alpha_{\tau'\pi}\beta_{\tau'} & (\pi\in\mathcal{P}^p,\ \tau\in\mathcal{T}_\pi^+,\ \tau'\in\mathcal{T}_\pi^-) & (6)\\
& \sum_{\tau\in\mathcal{T}} \varepsilon_\tau \le \overline{\varepsilon} & & (7)\\
& \varepsilon_\tau \in \mathbb{Z}_{\ge 0} & (\tau\in\mathcal{T}) & (8)\\
& \xi \in \mathbb{Z}_{\ge 0} & & (9)
\end{cases}
$$

4 Cyclic batch scheduling and concatenation

- Determine feasible subschedule $S' = (S_1, \ldots, S_n)$ for the $n = \sum_{\tau \in \mathcal{T}} \varepsilon_\tau$ operations of one cycle with appropriate scheduling method
- Subschedule S' induces precedence relationships
  - ▷ between operations i, j being processed on the same unit: $i \preceq j$ if $S_j \geq S_i + p_i + c_{ij}$
  - ▷ between operations i producing and operations j consuming the same product: $i \preceq j$ if $S_j \geq S_i + p_i$
  - ▷ between operations i consuming and operations j producing the same product: $i \preceq j$ if $S_j \geq S_i - p_j$
- Precedence relationships preserve schedule feasibility when moving operations

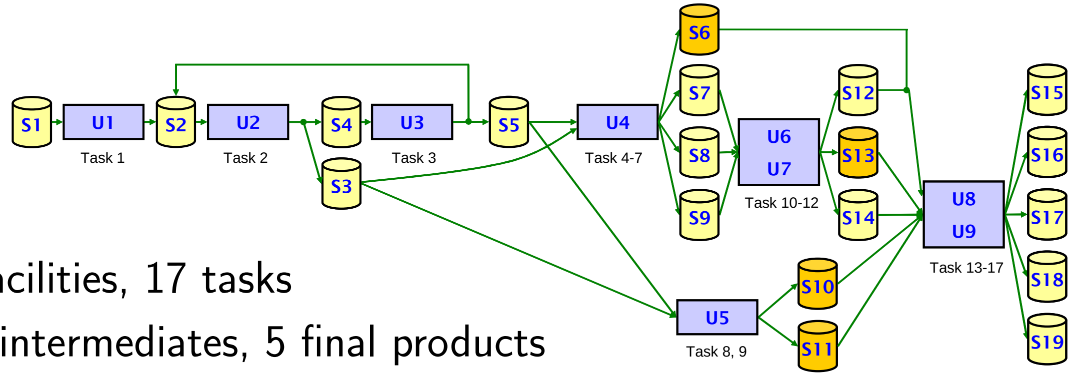


- Iteratively concatenate ξ "flexible" copies of subschedule S'
- Schedule S for first μ cycles defines release dates r_j for operations j of cycle $\mu + 1$
  - ▷ last operation i on unit in cycle μ and first operation j one that unit in cycle $\mu + 1$: $r_j = S_i + p_i + c_{ij}$
  - ▷ last operation i producing intermediate in cycle μ and operations j consuming that intermediate in cycle $\mu + 1$: $r_j = S_i + p_i$
  - ▷ last operation i consuming intermediate in cycle μ and operations j producing that intermediate in cycle $\mu + 1$: $r_j = S_i - p_j$
- Translate precedence relationships and release dates into prescribed minimum and maximum time lags between operations
- Temporal scheduling problem
  - ▷ Minimize makespan subject to minimum and maximum time lags
  - ▷ Can be solved efficiently by finding longest path lengths in operation-on-node network



5 Performance analysis

Test bed



- Case study of Kallrath (2002)
  - ▷ 9 processing units, 9 storage facilities, 17 tasks
  - ▷ 13 intermediates, 4 perishable intermediates, 5 final products
- 5 x 14 instances generated by varying primary requirements for final products

Settings

- Cyclic batching: $\overline{\varepsilon} = 150$, Frontline Systems Solver Package
- Cyclic batch scheduling: priority-rule based method, implemented in C++
- Concatenation: label-correcting algorithm, implemented in C++
- PC: 3.4 GHz Pentium IV CPU, 1 GB RAM

Benchmark

- Decomposition method proposed by Gentner et al. (2004)
- Results reported by Gentner (2005); PC: 1.4 GHz Pentium IV CPU, 1 GB RAM

Computational results

Instance	Gentner (2005)		This paper		
	$C_{\max}$	t_{cpu}	# op.'s	$C_{\max}$	t_{cpu}
WeKa0_0	178	18	88	128	116
WeKa0_1	352	38	176	252	113
WeKa0_2	474	53	264	376	118
WeKa0_3	612	120	352	500	119
WeKa0_4	738	209	440	624	115
WeKa0_5	906	178	528	748	122
WeKa0_6	1046	215	616	872	119
WeKa0_7	1199	323	704	996	121
WeKa0_8	1334	281	792	1120	117
WeKa0_9	1548	399	880	1244	128
WeKa0_10	1740	431	968	1368	100
WeKa0_15	2123	644	1408	1988	97
WeKa0_20	2899	1500	1848	2608	97
WeKa0_30	4416	5235	2728	3884	77
WeKa19_0	238	19	105	166	80
WeKa19_1	436	165	210	316	81
WeKa19_2	618	59	315	466	79
WeKa19_3	818	97	420	616	80
WeKa19_4	1004	179	525	766	81
WeKa19_5	1184	232	630	916	80
WeKa19_6	1384	330	735	1066	83
WeKa19_7	1570	474	840	1216	81
WeKa19_8	1806	442	945	1366	81
WeKa19_9	1946	568	1050	1516	80
WeKa19_10	2135	570	1155	1666	83
WeKa19_15	2848	1322	1680	2416	79
WeKa19_20	3811	1911	2205	3166	78
WeKa19_30	5896	6610	3255	4666	76
WeKa20_0	168	34	89	138	86
WeKa20_1	336	50	178	264	87
WeKa20_2	590	72	267	390	90
WeKa20_3	750	76	356	516	100
WeKa20_4	896	93	445	642	98
WeKa20_5	990	126	534	768	100
WeKa20_6	1138	184	623	894	95

Instance	Gentner (2005)		This paper		
	$C_{\max}$	t_{cpu}	# op.'s	$C_{\max}$	t_{cpu}
WeKa20_7	1294	215	712	1020	95
WeKa20_8	1547	200	801	1146	96
WeKa20_9	1816	327	890	1272	94
WeKa20_10	1920	448	979	1398	94
WeKa20_15	2386	421	1424	2028	96
WeKa20_20	3604	969	1869	2658	96
WeKa20_30	5194	3255	2759	3918	75
WeKa21_0	210	17	98	144	103
WeKa21_1	382	127	196	284	100
WeKa21_2	555	67	294	424	95
WeKa21_3	728	97	392	564	91
WeKa21_4	868	152	490	704	86
WeKa21_5	1082	226	588	844	86
WeKa21_6	1224	250	686	984	83
WeKa21_7	1420	240	784	1124	82
WeKa21_8	1554	291	882	1264	85
WeKa21_9	1701	475	980	1404	85
WeKa21_10	1916	469	1078	1544	82
WeKa21_15	2545	771	1568	2244	81
WeKa21_20	3398	1415	2058	2944	82
WeKa21_30	5091	5957	3038	4344	89
WeKa22_0	190	192	102	152	327
WeKa22_1	376	85	204	290	644
WeKa22_2	558	102	306	428	298
WeKa22_3	722	120	408	566	155
WeKa22_4	930	249	510	704	239
WeKa22_5	1024	239	612	842	324
WeKa22_6	1298	255	714	980	270
WeKa22_7	1488	341	816	1118	150
WeKa22_8	1520	439	918	1256	276
WeKa22_9	1779	427	1020	1394	149
WeKa22_10	1786	647	1122	1532	221
WeKa22_15	2586	704	1632	2222	171
WeKa22_20	3172	1598	2142	2912	206
WeKa22_30	5375	7563	3162	4292	271

6 Conclusions

Summary

- Short-term planning of batch production in the process industries
- New heuristic solution method for large-scale problem instances
  - ▷ Cyclic batching
  - ▷ Cyclic batch scheduling
  - ▷ Concatenation

Further research

- Expansion to continuous production mode
  - ▷ Production rates and processing times subject to optimization
- Coping with uncertainty: processing times, yields
  - ▷ Reactive scheduling methods for schedule revision and rescheduling