The preemptive project scheduling problem with generalized precedence relationships

New Challenges in Scheduling Theory 2012, Fréjus

Christoph Schwindt

Operations Management Group
Clausthal University of Technology

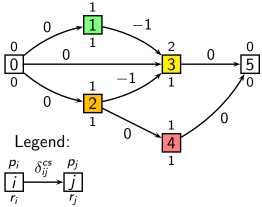
October 22th, 2012

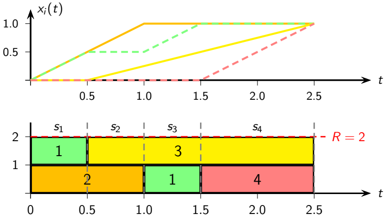
TU Clausthal
Clausthal University of Technology

Outline

Problem formulation

- Project consists of n activities $i \in V$ with durations $p_i \in \mathbb{Z}_{\geq 0}$
- Activities i use $r_{ik} \in \mathbb{Z}_{\geq 0}$ units of renewable resources $k \in \mathcal{R}$ with capacities $R_k \in \mathbb{Z}_{\geq 0}$ during their execution
- Each activity may be interrupted at any point in time
- For activity pairs $(i,j) \in E$, generalized precedence relationships $\Delta_{ij} = (\xi_i, \xi_j, \delta_{ij})$ are given: relative progress of activity j must not exceed percentage ξ_j earlier than δ_{ij} time units after activity i attained progress percentage ξ_i
- Project duration is to be minimized





Selected literature

Słowiński R (1980)
Two approaches to problems of resource allocation among project activities: A comparative study
The Journal of the Operational Research Society 31:711–723

Demeulemeester E, Herroelen W (1996)
An efficient optimal solution procedure for the preemptive resource-constrained project scheduling problem
European Journal of Operational Research 90:334–348

Damay J, Quilliot A, Sanlaville E (2007)
Linear programming based algorithms for preemptive and non-preemptive RCPSP
European Journal of Operational Research 182:1012–1022

Ballestin F, Valls V, Quintanilla S (2008)
Pre-emption in resource-constrained project scheduling
European Journal of Operational Research 189:1136–1152

Generalized precedence relationships

Relative progress of activity j must not exceed percentage ξ_j earlier than δ_{ij} time units after activity i attained progress percentage ξ_i

Formal statement of $\Delta_{ij} = (\xi_i, \xi_j, \delta_{ij})$ with $0 < \xi_i \leq 1$ and $0 \leq \xi_j < 1$

$$
\begin{aligned}
& t' < \min\{t | x_i(t) = \xi_i\} + \delta_{ij} \Rightarrow x_j(t') \leq \xi_j \\
\Leftrightarrow \quad & x_j(t') > \xi_j \Rightarrow t' \geq \min\{t | x_i(t) = \xi_i\} + \delta_{ij} \\
\Leftrightarrow \quad & \inf\{t | x_j(t) > \xi_j\} \geq \min\{t | x_i(t) = \xi_i\} + \delta_{ij} \\
\Leftrightarrow \quad & \max\{t | x_j(t) = \xi_j\} \geq \min\{t | x_i(t) = \xi_i\} + \delta_{ij} \\
\Leftrightarrow \quad & t_j^+(\xi_j) \geq t_i^-(\xi_i) + \delta_{ij}
\end{aligned}
$$

- Ordinary precedence constraints: $\Delta_{ij} = (1, 0, 0)$
- Completion-to-start minimum time lags: $\Delta_{ij} = (1, 0, \delta_{ij})$
- Maximum time lags: $\delta_{ij} < 0$

Descriptive model

Define binary function

$$y_i(t) := p_i \frac{d^+ x_i}{dt}(t) = \begin{cases} 1, \text{ if } i \text{ is in progress at time } t \\ 0, \text{ otherwise} \end{cases}$$

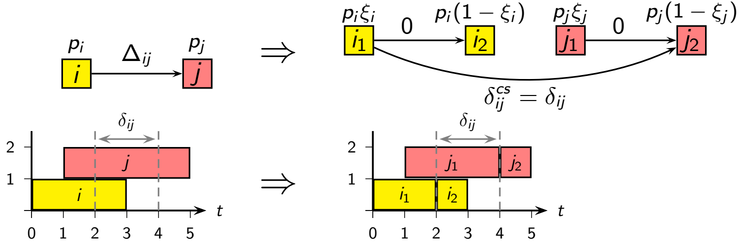
Model for problem $PS|pmtn, temp|C_{\max}$

$$(P) \quad \begin{cases} \text{Minimize} & C_{\max} \\ \text{subject to} & C_{\max} \geq t_i^-(1) \quad (i \in V) \\ & \sum_{i \in V} r_{ik} y_i(t) \leq R_k \quad (k \in \mathcal{R};\ t \geq 0) \\ & t_j^+(\xi_j) \geq t_i^-(\xi_i) + \delta_{ij} \quad ((i,j) \in E) \end{cases}$$

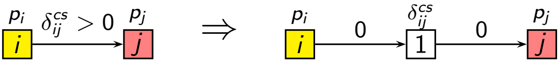
Canonical form of the problem

Without loss of generality we may assume that ...

- ... all generalized precedence relationships $\Delta_{ij} = (\xi_i, \xi_j, \delta_{ij})$ are specified as completion-to-start time lags $\Delta'_{ij} = (1, 0, \delta^{cs}_{ij})$



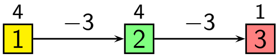
- ... all completion-to-start time lags δ^{cs}_{ij} are nonpositive

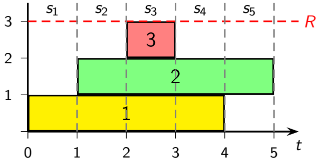


Number of slices

Proposition

1. For any feasible instance of problem $PS|pmtn, prec|C_{\max}$ there exists an optimal schedule x with at most n slices of positive duration.
2. For any feasible instance of problem $PS|pmtn, temp|C_{\max}$ in canonical form there exists an optimal schedule x with at most $2n-1$ slices of positive duration.





A new MILP formulation

MILP model for problem $PS|pmtn, temp|C_{\max}$

$$
(MILP) \quad \begin{cases}
\text{Minimize} & C_{\max} = \sum_{s \in \mathcal{S}} p_s & \\
\text{subject to} & 0 \leq p_s - p_i \Delta x_{is} \leq \bar{p} \cdot (1 - y_{is}) & (i \in V;\ s \in \mathcal{S}) \\
& 0 \leq \Delta x_{is} \leq y_{is} & (i \in V;\ s \in \mathcal{S}) \\
& \sum_{s \in \mathcal{S}} \Delta x_{is} = 1 & (i \in V) \\
& \sum_{i \in V} r_{ik} \cdot y_{is} \leq R_k & (k \in \mathcal{R};\ s \in \mathcal{S}) \\
& S_i \leq \sum_{s'=1}^{s-1} p_{s'} + \bar{d} \cdot (1 - y_{is}) & (i \in V;\ s \in \mathcal{S}) \\
& C_i \geq \sum_{s'=1}^{s} p_{s'} - \bar{d} \cdot (1 - y_{is}) & (i \in V;\ s \in \mathcal{S}) \\
& S_j \geq C_i + \delta_{ij}^{cs} & ((i,j) \in E) \\
& y_{is} \in \{0, 1\} & (i \in V;\ s \in \mathcal{S})
\end{cases}
$$

- $y_{is} = 1$: activity i executed in slice s
- p_s: duration of slice s
- Δx_{is}: increase in relative progress of activity i in slice s $(= \frac{p_s}{p_i} \cdot y_{is})$
- S_i: lower bound on start time of activity i
- C_i: upper bound on completion time of activity i

Computational results MILP model

- PC with 3.16 GHz and 3 GB RAM operating under Windows XP
- Model (*MILP*) coded under GAMS 23.7 invoking CPLEX 12.0 as MILP solver
- Solver stopped after a time limit of 300 seconds

Performance of the MILP model for the KSD-30 instances

RS	p_{term}	p_{opt}	Δ_{pmtn}	Δ_{nonp}	Δ_{nonp}^{min}	p_{imp}
0.2	10.0 %	38.3 %	2.5 %	−0.7 %	−8.8 %	43.3 %
0.5	38.3 %	64.2 %	0.9 %	−1.5 %	−6.9 %	55.8 %
0.7	80.0 %	89.2 %	0.4 %	−0.7 %	−6.7 %	26.7 %
1.0	100.0 %	100.0 %	0.0 %	0.0 %	0.0 %	0.0 %
Total	57.1 %	72.9 %	0.9 %	−0.7 %	−8.8 %	30.4 %

Performance of the MILP model for the UBO-10 instances

RS	p_{opt}	p_{inf}	p_{feas}	p_{unk}	Δ_{nonp}	Δ_{nonp}^{min}	p_{imp}	$\#_{pmtn}$
0.0	20.0 %	20.0 %	46.7 %	13.3 %	−0.8 %	−5.4 %	16.7 %	5.14
0.25	33.3 %	6.7 %	46.7 %	13.3 %	−2.2 %	−16.7 %	33.3 %	5.50
0.5	56.7 %	13.3 %	30.0 %	0.0 %	−1.5 %	−10.9 %	33.3 %	5.54
Total	36.7 %	13.3 %	41.1 %	8.9 %	−1.5 %	−16.7 %	27.8 %	5.41

Computational results MILP model

- PC with 3.16 GHz and 3 GB RAM operating under Windows XP
- Model (*MILP*) coded under GAMS 23.7 invoking CPLEX 12.0 as MILP solver
- Solver stopped after a time limit of 300 seconds

Performance of the MILP model for the KSD-30 instances

RS	p_{term}	p_{opt}	Δ_{pmtn}	Δ_{nonp}	Δ_{nonp}^{min}	p_{imp}
0.2	10.0 %	38.3 %	2.5 %	−0.7 %	−8.8 %	43.3 %
0.5	38.3 %	64.2 %	0.9 %	−1.5 %	−6.9 %	55.8 %
0.7	80.0 %	89.2 %	0.4 %	−0.7 %	−6.7 %	26.7 %
1.0	100.0 %	100.0 %	0.0 %	0.0 %	0.0 %	0.0 %
Total	57.1 %	72.9 %	0.9 %	−0.7 %	−8.8 %	30.4 %

Performance of the MILP model for the UBO-10 instances

RS	p_{opt}	p_{inf}	p_{feas}	p_{unk}	Δ_{nonp}	Δ_{nonp}^{min}	p_{imp}	$\#_{pmtn}$
0.0	20.0 %	20.0 %	46.7 %	13.3 %	−0.8 %	−5.4 %	16.7 %	5.14
0.25	33.3 %	6.7 %	46.7 %	13.3 %	−2.2 %	−16.7 %	33.3 %	5.50
0.5	56.7 %	13.3 %	30.0 %	0.0 %	−1.5 %	−10.9 %	33.3 %	5.54
Total	36.7 %	13.3 %	41.1 %	8.9 %	−1.5 %	−16.7 %	27.8 %	5.41

Computational results MILP model

- PC with 3.16 GHz and 3 GB RAM operating under Windows XP
- Model (*MILP*) coded under GAMS 23.7 invoking CPLEX 12.0 as MILP solver
- Solver stopped after a time limit of 300 seconds

Performance of the MILP model for the KSD-30 instances

RS	p_{term}	p_{opt}	Δ_{pmtn}	Δ_{nonp}	Δ_{nonp}^{min}	p_{imp}
0.2	10.0 %	38.3 %	2.5 %	−0.7 %	−8.8 %	43.3 %
0.5	38.3 %	64.2 %	0.9 %	−1.5 %	−6.9 %	55.8 %
0.7	80.0 %	89.2 %	0.4 %	−0.7 %	−6.7 %	26.7 %
1.0	100.0 %	100.0 %	0.0 %	0.0 %	0.0 %	0.0 %
Total	57.1 %	72.9 %	0.9 %	−0.7 %	−8.8 %	30.4 %

Performance of the MILP model for the UBO-10 instances

RS	p_{opt}	p_{inf}	p_{feas}	p_{unk}	Δ_{nonp}	Δ_{nonp}^{min}	p_{imp}	$\#_{pmtn}$
0.0	20.0 %	20.0 %	46.7 %	13.3 %	−0.8 %	−5.4 %	16.7 %	5.14
0.25	33.3 %	6.7 %	46.7 %	13.3 %	−2.2 %	−16.7 %	33.3 %	5.50
0.5	56.7 %	13.3 %	30.0 %	0.0 %	−1.5 %	−10.9 %	33.3 %	5.54
Total	36.7 %	13.3 %	41.1 %	8.9 %	−1.5 %	−16.7 %	27.8 %	5.41

Continuous model

- Consider all feasible antichains $\mathcal{A} \in \mathcal{F}$
- Antichains $\mathcal{A}, \mathcal{A}' \in \mathcal{F}$ ordered ($\mathcal{A} \to \mathcal{A}'$) if
  - $d_{ij}^{cs} \geq 0$ for some $i \in \mathcal{A},\ j \in \mathcal{A}'$ or
  - $\mathcal{A} \to \mathcal{A}''$ and $\mathcal{A}'' \to \mathcal{A}'$ for some $\mathcal{A}'' \in \mathcal{F}$
- $\mathcal{D} \subseteq \mathcal{F}$ incompatibility set if $|\mathcal{D}| \geq 2$ and $\mathcal{A} \to \mathcal{A}'$, $\mathcal{A}' \to \mathcal{A}$ for all $\mathcal{A}, \mathcal{A}' \in \mathcal{D} : \mathcal{A} \neq \mathcal{A}'$

Linear program with incompatibility constraints

$$(LPCC) \quad \left\{ \begin{array}{lll} \text{Minimize} & C_{\max} = \sum_{\mathcal{A}} p_{\mathcal{A}} & \\ \text{subject to} & \sum_{\mathcal{A}: i \in \mathcal{A}} p_{\mathcal{A}} = p_i & (i \in V) \\ (*) & \prod_{\mathcal{A} \in \mathcal{D}} p_{\mathcal{A}} = 0 & (\text{incompatibility sets } \mathcal{D}) \\ & p_{\mathcal{A}} \geq 0 & (\mathcal{A} \in \mathcal{F}) \end{array} \right.$$

- Model $(LPCC)$
  - exact for $PS|pmtn, prec|C_{\max}$
  - relaxation for $PS|pmtn, temp|C_{\max}$

Column generation

Lower and upper bounds

- ($LPCC$) without constraints ($*$) is linear program (LP) with huge number of decision variables
- (LP) can be solved efficiently by column generation: lower bounds
- Pricing problem corresponds to multi-dimensional knapsack problem
- Generate feasible, locally optimal schedules for $PS|pmtn, prec|C_{\max}$ by maintaining condition ($*$) during pivoting: upper bounds (method of Damay et al. 2007)

Performance of the column generation procedures

KSD-30	Δ_{opt}^{LB}	n_{it}	t_{cpu}	Δ_{opt}^{UB}	p_{term}	p_{opt}	n_{it}	t_{cpu}
	2.05 %	73.0	11.9 s	1.97 %	49.4 %	62.9 %	76.2	23.7 s
UBO-10	Δ_{best}^{LB}	n_{it}	t_{cpu}					
	3.03 %	47.1	3.6 s					

Column generation and Variable Neighborhood Descent VND

- Challenges in computing upper bounds via column generation when project network is cyclic
  - finding first feasible schedule is NP-hard
  - checking feasibility of given basis is NP-complete (transformation from $1|pmtn, temp|C_{\max}$)
- Generate first feasible schedule by dualizing precedence relationships (i, j) with $\delta_{ij}^{cs} < 0$ (dual model $\overline{MILP}$)
- Compute improving nonbasic antichain by solving model MILP(ℓ) where binaries y_{is} can be modified for exactly $\ell = 1$ slice s
- $MILP(\ell)$ results from $MILP$ by adding two simple constraints

$$(1 - \hat{y}_{is}) \cdot y_{is} + \hat{y}_{is} \cdot (1 - y_{is}) \leq z_s \quad (i \in V; s \in \mathcal{S})$$
$$\sum_{s \in \mathcal{S}} z_s \leq \ell$$

- Allowing for $\ell > 1$: Variable Neighborhood Descent heuristic

Algorithm 1 Variable Neighborhood Descent

Input: instance of $PS|pmtn, temp|C_{\max}$, max. neighborhood dimension $\overline{\ell}$
Output: feasible schedule

determine feasible schedule by solving dual $\overline{MILP}$ to optimality;
put $\ell := 1$ and stop := false;
while $\neg$ stop **do**
  solve model $MILP(\ell)$;
  if $C_{\max}$ has been improved **then** put $\ell := 1$;
  elsif $\ell = \infty$ **then** stop := true;
  else
    put $\ell := \ell + 1$;
    if $\ell > \overline{\ell}$ **then** put $\ell := \infty$;
  end if
end while

Computational results VND

- Maximum neighborhood dimension $\overline{\ell} = 3$
- CPU time limits
  - Dual model $\overline{MILP}$: 300 seconds
  - Neighborhood search models $MILP(\ell)$: 30 seconds

Performance of VND heuristic for the UBO-10 instances

RS	p_{opt}	p_{inf}	p_{feas}	p_{unk}	Δ_{nonp}	n_{it}	t_{cpu}
0.0	20.0 %	20.0 %	60.0 %	0.0 %	−0.64 %	7.8	71.7 s
0.25	40.0 %	10.0 %	50.0 %	0.0 %	−1.74 %	8.9	88.8 s
0.5	50.0 %	13.3 %	36.7 %	0.0 %	−1.26 %	11.1	104.4 s
Total	36.7 %	14.4 %	48.9 %	0.0 %	−1.59 %	9.3	88.3 s

- All eight previously open instances solved to feasibility
- Four of those instances infeasible when preemption is not allowed
- 44 % less CPU time than MILP model (158.2 seconds)
- Slightly larger improvement on nonpreemptive solutions

Summary and future work

Summary

- Preemptive project scheduling with generalized precedence relationships
- Compact descriptive model and MILP formulation
- Lower and upper bounds by column generation and VND heuristic

Future work

1. Decomposition methods for larger instances of $PS|pmtn, temp|C_{\max}$
2. Branch-and-bound algorithm for $PS|pmtn, temp|C_{\max}$ resolving incompatibilities in solutions to LP relaxation of continuous model

Open questions:

- Upper bound on number of preemptions ($< (2n-1) \cdot \lfloor \frac{n-1}{2} \rfloor$?)
- Maximum rel. improvement by preemption ($> 1 - \frac{1}{\max_k R_k + 1}$?)

References

Ballestin F, Valls V, Quintanilla S (2008)
Pre-emption in resource-constrained project scheduling
European Journal of Operational Research 189:1136–1152

Damay J, Quilliot A, Sanlaville E (2007)
Linear programming based algorithms for preemptive and non-preemptive RCPSP
European Journal of Operational Research 182:1012–1022

Demeulemeester E, Herroelen W (1996)
An efficient optimal solution procedure for the preemptive resource-constrained project scheduling problem
European Journal of Operational Research 90:334–348

Franck B, Neumann K, Schwindt C (2001)
Truncated branch-and-bound, schedule-construction, and schedule-improvement procedures for resource-constrained project scheduling
OR Spektrum 23:297–324

Kaplan L (1988)
Resource-constrained project scheduling with preemption of jobs
Unpublished PhD Dissertation, University of Michigan

References

Kolisch R, Sprecher A (1996)
PSPLIB: A project scheduling library
European Journal of Operational Research 99:205–216

Nadjafi BA, Shadrokh S (2008)
The preemptive resource-constrained project scheduling problem subject to due dates and preemption penalties: An integer programming approach
Journal of Industrial Engineering 1:35–39

Neumann K, Schwindt C, Zimmermann J (2003)
Project Scheduling with Time Windows and Scarce Resources, 2nd ed.
Springer, Berlin

Richter LK, Yano CA (1986)
A comparison of heuristics for preemptive resource-constrained project scheduling
Technical Report, 86-39, University of Michigan, Ann Arbor

Słowiński R (1980)
Two approaches to problems of resource allocation among project activities: A comparative study
The Journal of the Operational Research Society 31:711–723

Ordinary precedence constraints

Alternative formulation of precedence relationships $\Delta_{ij} = (1, 0, 0)$

$$(|\mathcal{S}| - s + 1) \cdot y_{js} \leq |\mathcal{S}| - s + 1 - \sum_{s'=s}^{|\mathcal{S}|} y_{is'} \quad (s \in \mathcal{S})$$

Strengthening the LP relaxation

Proportion variables

$$p_i \Delta x_{is} \geq p_j \Delta x_{js} - p_j \cdot (1 - y_{is}) \quad (i, j \in V : i \neq j)$$
$$p_s \geq \frac{1}{R_k} \cdot \sum_{i \in V} r_{ik} p_i \Delta x_{is} \quad (k \in \mathcal{R};\ s \in \mathcal{S})$$

Tail- and head-based upper bounds

$$C_{\max} \geq \sum_{s'=1}^{s} (p_{s'} - p_i \Delta x_{is'}) + p_i + q_i y_{is} \quad (i \in V;\ s \in \mathcal{S})$$
$$C_{\max} \geq r_i y_{is} + p_i + \sum_{s'=s}^{|\mathcal{S}|} (p_{s'} - p_i \Delta x_{is'}) \quad (i \in V;\ s \in \mathcal{S})$$

Disjunctive activities

$$y_{is} + y_{js} \leq 1 \quad (i, j \in V : r_{ik} + r_{jk} > R_k \text{ for some } k \in \mathcal{R};\ s \in \mathcal{S})$$
$$y_{is} + y_{js} \leq 1 \quad (i, j \in V : d_{ij}^{cs} \geq 0;\ s \in \mathcal{S})$$