Local Search for Project Scheduling with Convex Objective Functions

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Abstract

We consider the scheduling of projects subject to temporal and resource constraints such that a continuous convex objective function in the start times of activities is minimized. Optimal solutions to this problem represent local minimizers on polytopes belonging to inclusion–minimal feasible orders which reflect precedence relationships between activities. Thus, a natural approach to the problem is to enumerate appropriate orders for each of which a local minimizer of the objective function on the corresponding polytope is determined by a descent algorithm. We propose different neighborhoods on the set of orders which arise from removing, replacing, or adding pairs in the respective covering relation. For the resource–constrained weighted earliness–tardiness problem, a corresponding tabu search procedure has been implemented which uses a primal and a dual first–order descent algorithm for the computation of stationary points.

1 Preliminaries

Let $N = \langle V, E; \delta\rangle$ be an activity–on–node project network with set of nodes V, set of arcs E, and corresponding arc weights δ. By $\mathcal{R}$ we denote the set of resources required for the execution of the project. The start times S_i of activities $i \in V$ have to be determined such that a continuous convex objective function f of schedule $S = (S_i)_{i\in V}$ is minimized, the temporal constraints $S_j \geq S_i + \delta_{ij}$ belonging to arcs $\langle i, j\rangle \in E$ are met, the project is started at time zero and completed by a deadline $\overline{d}$, and the requirements $r_k(S, t)$ of resources $k \in \mathcal{R}$ do not exceed the corresponding resource capacities R_k at any point in time t. This problem, designated as $PS|temp, \overline{d}|f$ and $m, 1|gpr, \delta_n|nonreg$ in the triple classifications devised by Brucker et al. (1999) and Herroelen et al. (1998), respectively, can be stated as follows:

$$\left\{\begin{array}{lll} \text{Minimize} & f(S) & \quad (1.1)\\ \text{subject to} & S_j - S_i \geq \delta_{ij} \quad (\langle i,j\rangle \in E) & \quad (1.2)\\ & S_i \geq 0 \quad (i \in V) & \quad (1.3)\\ & S_0 = 0, S_{n+1} \leq \overline{d} & \quad (1.4)\\ & r_k(S,t) \leq R_k \quad (k \in \mathcal{R}, t \geq 0) & \quad (1.5)\end{array}\right.$$

Let $\mathcal{S}_T$ be the set of time–feasible schedules observing the temporal constraints (1.2), (1.3), and (1.4). The feasible region $\mathcal{S}$ is the set of time–feasible schedules complying with the resource constraints (1.5). Schedules $S \in \mathcal{S}$ are referred to as feasible schedules. Optimal schedules are feasible schedules minimizing f.

Let O be a strict order in set V. By $\mathcal{S}(O)$ we denote the corresponding order polytope (cf. Neumann, 2000). $\mathcal{S}(O)$ represents the set of all time–feasible schedules meeting the precedence constraints $S_j \geq S_i + p_i$ for all $(i, j) \in O$, where p_i denotes the duration of activity i. O is called feasible if $\emptyset \neq \mathcal{S}(O) \subseteq \mathcal{S}$.

The feasible region $\mathcal{S}$ corresponds to the union of polytopes $\mathcal{S}(O)$ belonging to feasible orders. For $\mathcal{S} \neq \emptyset$, there is always an inclusion–minimal feasible order O such that all local minimizers of f on $\mathcal{S}(O)$ represent optimal schedules. The computation of these local minimizers can be performed by descent algorithms. In Section 2 we propose three different neighborhoods generating (covering relations of) appropriate orders O.

2 Local Search

We represent orders O by corresponding covering relations ρ. The neighborhoods are defined on the set of covering relations ρ of orders O which are given by binding precedence constraints. Let S be a schedule minimizing f on some order polytope $\mathcal{S}(O')$. The corresponding relation ρ is then given as covering relation of the order $O \subseteq O'$ which is induced by pairs (i, j) with $S_j = S_i + p_i$. Clearly, S minimizes f on $\mathcal{S}(O) \supseteq \mathcal{S}(O')$ as well.

For relation ρ, let $P(\rho)$ denote the problem of minimizing f on $\mathcal{S}(O)$. Relation ρ can possibly be reduced as follows. We determine an optimal solution S' to problem $P(\rho \setminus \{(i, j)\})$ for each pair $(i, j) \in \rho$ in order of nonincreasing values of dual variables u_{ij} of the steepest descent problem at S. If $S' = S$ or if S' is feasible, we remove pair (i, j) from relation ρ and set S to be S'. ρ then defines a set of precedence constraints which all are necessary for settling resource conflicts.

The first neighborhood can now be described as follows. First, we determine an optimal solution S' to all problems $P(\rho \setminus \{(i, j)\})$ with $(i, j) \in \rho$ and define respective neighbors ρ' of ρ as the sets of pairs (v, w) with $S'_w - S'_v = p_v$. For all schedules S' and, if S is infeasible for S, we then determine the earliest point in time t at which the resource constraints (1.5) are violated and the corresponding set $\mathcal{A}(S', t)$ or $\mathcal{A}(S, t)$, respectively, of activities processed at time t. For all $g, h \in \mathcal{A}(S', t)$ with $g \neq h$ and $(g, h) \neq (i, j)$, we consider $P(\rho \setminus \{(i, j)\} \cup \{(g, h)\})$ and for all $g, h \in \mathcal{A}(S, t)$ with $g \neq h$, we consider $P(\rho \cup \{(g, h)\})$. If the problem is solvable, we determine an optimal solution S'', and the corresponding neighbor ρ'' of ρ is the set of pairs (v, w) with $S''_w - S''_v = p_v$. The maximum number of neighbors of ρ is of order $|\rho| n^2$.

A second neighborhood, which may be used for diversification, arises from choosing (g, h) to be (j, i). The maximum number of neighbors is then of order n^2. By choosing

(g,h) to be (i,h) or (g,j) for some activities h or g which are executed simultaneously with activity i or j, respectively, we obtain a third neighborhood for intensification with maximum cardinality of order $\max(|\rho|n, n^2)$.

Generally, optimal solutions to problems $P(\rho)$ do not represent feasible schedules. That is why we measure the quality of schedules S by cost function

$$f^c(S) = f(S) + \lambda \sum_{k \in \mathcal{R}} \int_0^{\overline{d}} \max(0, r_k(S,t) - R_k)\, dt$$

with $\lambda > 0$ which penalizes violations of the resource constraints (1.5).

Based on the presented neighborhoods we implemented a tabu search algorithm for the weighted resource–constrained earliness–tardiness problem with objective function

$$f(S) = \sum_{i \in V} (w_i^- \max(0, d_i - S_i - p_i) + w_i^+ \max(0, S_i + p_i - d_i))$$

where w_i^-, w_i^+, and d_i represent the earliness cost, the tardiness cost, and the due date, respectively, of activity i (cf. Vanhoucke et al., 1999 for a recent branch–and–bound algorithm for the case where $\delta_{ij} = p_i$ for all $\langle i,j \rangle \in E$). For solving problems $P(\rho)$, we use a primal and a dual algorithm. The corresponding first–order steepest descent problems can be solved in linear order time if the direction is normalized by the supremum norm (cf. Schwindt, 1999). Computational experience indicates that for this resource–constrained project scheduling problem, local search algorithms are competitive with truncated versions of a branch–and–bound procedure described in Schwindt (1999).

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