Scheduling of Continuous Production in Process Industries

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Abstract

We consider the scheduling of a chemical multi-level production plant processing a given set of operations. The production levels are decoupled by intermediate storage facilities of finite capacity. For each operation the processing time, the requirements for processing units and manpower, as well as the total quantities of input products consumed and output products produced are known. The problem consists of allocating processing units and manpower over time such that at any point in time, a sufficient amount of input products and sufficient storage capacity for output products is available, prescribed time lags between operations are observed, and some convex objective function in the start times of the operations is minimized. We propose a branch-and-bound algorithm that is based on the representation of resource constraints as disjunctions of linear inequality systems.

1. Introduction

Let $O = \{0, 1, \ldots, n, n+1\}$ be a set of operations to be executed on a chemical production plant, where 0 and $n+1$ denote two fictitious operations representing the production start and the production termination, respectively. The plant consists of processing units that are linked by storage facilities for intermediate products. Besides a processing unit, each operation requires manpower, input products, and storage space for output products. An operation may be processed in batch or in continuous production mode. We speak of batch production if the input products are consumed at the start and the output product arise at the completion of the operation. An operation is processed in continuous production mode if input products are consumed and output products arise at constant rates.

Processing units and manpower represent renewable resources, whose availability is independent from previous utilization. Let $\mathcal{R}^\rho$ be the set of all renewable resources, and let R_k denote the capacity of resource $k \in \mathcal{R}^\rho$. Inventories of intermediate products stocked in storage facilities are modelled as cumulative resources, which are depleted and replenished over time (see Trautmann, 2002). For notational convenience we assume that all cumulative resources are either operated in batch access mode where all depletion and replenishment rates are infinite (batch cumulative resources) or operated in continuous access mode where all depletion and replenishment rates are finite (continuous cumulative resources). $\mathcal{R}^\beta$ and $\mathcal{R}^\gamma$ denote the sets of batch and continuous cumulative resources, respectively. For each cumulative resource $k \in \mathcal{R}^\beta \cup \mathcal{R}^\gamma$, a minimum inventory level or safety stock $\underline{R}_k$ and a maximum inventory level or storage capacity $\overline{R}_k$ are given.

Each operation $i \in O$ is associated with a processing time p_i and demands r_{ik} for resources $k \in \mathcal{R}^\rho \cup \mathcal{R}^\beta \cup \mathcal{R}^\gamma$. If k is a renewable resource, r_{ik} equals the number of units of resource k used for processing i. If k is a cumulative resource, r_{ik} is the total increase in the inventory level of resource k caused by operation i. If $r_{ik} < 0$, operation i depletes resource k by $-r_{ik}$ units, and if $r_{ik} > 0$, operation i replenishes resource k by r_{ik} units.

For certain pairs of operations (i, j) from a set $TL \subset O \times O$, a minimum time lag δ_{ij} between the start of operation i and the start of operation j is prescribed. A negative minimum time lag δ_{ij} can be interpreted as a positive maximum time lag $-\delta_{ij}$ between operation j and operation i. These temporal constraints may arise from technological or organizational requirements such as release dates for raw materials, delivery dates for final products, or quarantine and shelf life times for intermediate products (cf. Trautmann, 2002).

We consider the problem of finding a start time S_i for each operation $i \in O$ such that some convex function f in start times S_i is minimized and the constraints arising from the limited availability of renewable and cumulative resources as well as the temporal constraints are taken into account.

2. Model

Let $S = (S_i)_{i \in O}$ with $S_0 = 0$ and $S_i \geq 0$ for all $i \in O$ be some (production) schedule. $\mathcal{A}(S,t) = \{i \in O \mid S_i \leq t < S_i + p_i\}$ is the set of all operations i being in progress at time t, and $r_k^\rho(S,t) = \sum_{i \in \mathcal{A}(S,t)} r_{ik}$ is the amount of renewable resource k used at time t. For given cumulative resource k, let $O_k^- = \{i \in O \mid r_{ik} < 0\}$ and $O_k^+ = \{i \in O \mid r_{ik} > 0\}$ denote the sets of operations depleting and replenishing, respectively, resource k. For batch cumulative resource $k \in \mathcal{R}^\beta$, $\mathcal{A}_k(S,t) = \{i \in O_k^- \mid S_i \leq t\} \cup \{i \in O_k^+ \mid S_i + p_i \leq t\}$ is the set of all operations that have depleted and replenished resource k by time t, and $r_k^\beta(S,t) = \sum_{i \in \mathcal{A}_k(S,t)} r_{ik}$ is the inventory level in resource k at time t. With

$$x_i(S,t) = \begin{cases} 0, \text{ if } t < S_i \\ 1, \text{ if } t \geq S_i + p_i \\ (t - S_i)/p_i, \text{ otherwise} \end{cases}$$

we denote the portion of operation $i \in O$ that has been processed by time t. The inventory in continuous cumulative resource $k \in \mathcal{R}^\gamma$ at time t is $r_k^\gamma(S,t) = \sum_{i \in O} r_{ik} x_i(S,t)$. The production scheduling problem (PSP) to be dealt with can be formulated as follows.

$$(\text{PSP}) \begin{cases} \text{Minimize} & f(S) & & \\ \text{subject to} & r_k^\rho(S,t) \leq R_k & (k \in \mathcal{R}^\rho,\ t \geq 0) & (1) \\ & \underline{R}_k \leq r_k^\beta(S,t) \leq \overline{R}_k & (k \in \mathcal{R}^\beta,\ t \geq 0) & (2) \\ & \underline{R}_k \leq r_k^\gamma(S,t) \leq \overline{R}_k & (k \in \mathcal{R}^\gamma,\ t \geq 0) & (3) \\ & S_j - S_i \geq \delta_{ij} & ((i,j) \in TL) & (4) \\ & S_0 = 0,\ S_i \geq 0 & (i \in O) & (5) \end{cases}$$

A schedule S satisfying the temporal constraints (4) and (5) is termed time-feasible. A feasible schedule is a time-feasible schedule S that complies with the resource constraints (1) to (3). S is an optimal schedule if it is feasible and minimizes f.

3. Solution Method

In what follows we expand an approach devised by De Reyck and Herroelen (1998) for project scheduling with renewable resources to deal with our problem. Laborie (2001) and Neumann (2002) discuss a generalization of the latter approach to scheduling problems with cumulative-resource constraints where the consumption and production rates are infinite.

The basic principle is as follows. We omit the resource constraints and solve the resulting convex program by computing some time-feasible schedule S that minimizes objective function f. If S is feasible, S is an optimal schedule. Otherwise, we determine some point in time t for which any of the resource constraints (1) to (3) is not satisfied. Depending on the type of resource constraint violated, we determine alternative sets of precedence relationships between operations resolving the resource conflict. We then select one alternative and add the corresponding precedence relationships to the convex program, which is then solved again. Solving the convex program and adding precedence relationships is repeated until either the feasible region becomes void or a feasible schedule has been found. In both cases we return to an alternative set of precedence relationships and proceed until all alternatives have been investigated.

In the following, we discuss how to to find appropriate precedence relationships between operations. We first briefly review the case where at time t, constraints (1) or (2) are violated (for details we refer to Neumann, 2002). Excessive utilization of renewable resources can be settled by introducing temporal constraints $S_j - S_i \geq p_i$ between operations i, j being processed at time t (i.e., $i, j \in \mathcal{A}(S,t)$). In case of an inventory shortage in a batch cumulative resource $k \in \mathcal{R}^\beta$ at time t, we add temporal constraints $S_j - S_i \geq p_i$ between replenishing operations i that are completed after time t (i.e., $i \in O_k^+ \setminus \mathcal{A}_k(S,t)$) and depleting operations j that have been started by time t (i.e., $j \in O_k^- \cap \mathcal{A}_k(S,t)$). If the storage capacity of resource k is exceeded at time t, we add temporal constraints $S_j - S_i \geq -p_j$ between depleting operations i that are started after time t (i.e., $i \in O_k^- \setminus \mathcal{A}_k(S,t)$) and replenishing operations j that have been completed by time t (i.e., $j \in O_k^+ \cap \mathcal{A}_k(S,t)$).

We now consider in more detail how to deal with violations of the resource constraints (3) referring to continuous cumulative resources. Assume that at time t, the inventory in some resource $k \in \mathcal{R}^\gamma$ falls below the safety stock $\underline{R}_k$. We partition the set $O_k = O_k^- \cup O_k^+$ of all operations depleting and replenishing resource k into two sets A and B with the following meaning. A contains all operations $j \in O_k^-$ that will have to be completed by deadline t and all operations $j \in O_k^+$ that will be released at time t, that is,

$$\left.\begin{array}{ll} S_j \leq t - p_j & (j \in A \cap O_k^-) \\ S_j \geq t & (j \in A \cap O_k^+) \end{array}\right\} \tag{6}$$

The depletion of resource k at time t arising from operations $j \in A$ equals $-\sum_{j \in A \cap O_k^-} r_{jk}$. The operations j from set B must be scheduled such that at time t, their cumulative replenishment of k is greater than or equal to the inventory shortage $\underline{R}_k - \sum_{j \in A \cap O_k^-} r_{jk}$ caused by the operations from set A. This can be achieved as follows. For each operation $j \in B$, we introduce a continuous decision variable x_j with

$$0 \leq x_j \leq 1 \quad (j \in B) \tag{7}$$

providing the portion of operation j that will be processed by time t. The requirement that the inventory in resource k at time t must not fall below $\underline{R}_k$ then reads

$$\sum_{j \in B} r_{jk} x_j \geq \underline{R}_k - \sum_{j \in A \cap O_k^-} r_{jk} \tag{8}$$

The coupling between decision variables x_j and S_j is achieved by the temporal constraints (parameterized in x_j)

$$\left.\begin{array}{ll} S_j \geq t - p_j x_j & (j \in B \cap O_k^-) \\ S_j \leq t - p_j x_j & (j \in B \cap O_k^+) \end{array}\right\} \tag{9}$$

Inequalities (9) imply that for each schedule S satisfying (9), $x_j \geq x_j(S,t)$ if operation $j \in B$ depletes and that $x_j \leq x_j(S,t)$ if operation $j \in B$ replenishes resource k. Adding constraints (6) to (9) to the convex program settles the inventory shortage at time t. Violations of the storage capacities can be dealt with analogously by interchanging the sets O_k^- and O_k^+ in inequalities (6) and (9) and replacing (8) by

$$\sum_{j \in B} r_{jk} x_j \leq \overline{R}_k - \sum_{j \in A \cap O_k^+} r_{jk} \tag{10}$$

The inventory in resource k attains its minimum at a point in time when some replenishing operation i is started or when some depleting operation i is completed. That is why time t can always be chosen to be equal to S_i for some $i \in O_k^+$ or equal to $S_i + p_i$ for some $i \in O_k^-$, and thus we can replace t in (6) and (9) by S_i or $S_i + p_i$. We then write A^{ik} and B^{ik} instead of A and B as well as x_j^{ik} instead of x_j. Note that without loss of generality we can assume $i \in A^{ik}$ for all $k \in \mathcal{R}^\gamma$ and all $i \in O_k$ because the corresponding inequality (6) is always satisfied. Switching from constants t to variables S_i ensures that only a finite number of constraints have to be introduced before constraints (3) are satisfied.

4. Implementationary Issues

Assume that the inventory in some resource $k \in \mathcal{R}^\gamma$ falls below the safety stock at time $t = S_i$ ($i \in O_k^+$) or $t = S_i + p_i$ ($i \in O_k^-$). To enumerate the sets A^{ik} and B^{ik} we construct a binary tree as follows. Each level of the tree belongs to one operation $j \in O_k$. For each j, we branch over the two alternatives $j \in A^{ik}$ and $j \in B^{ik}$ and add the respective constraints (6), (7), (9), and the relaxation

$$\sum_{j \in B^{ik}} r_{jk} x_j^{ik} \geq \underline{R}_k - \sum_{j \in A^{ik} \cap O_k^-} r_{jk} - \sum_{j \in O_k^+ \setminus A^{jk} \setminus B^{ik}} r_{jk} \tag{11}$$

of constraint (8) to the convex program. Each leaf of the tree corresponds to one distinct partition $\{A^{ik}, B^{ik}\}$. We can suspend the enumeration for pair (i,k) as soon as the inventory shortage at time S_i or $S_i + p_i$ is settled, even if $A^{ik} \cup B^{ik} \subset O_k$. In the latter case, it may be necessary to resume the branching later on if the shortage reappears while dealing with other resource conflicts. Since at most $3n(n-1)/2$ temporal constraints of type $S_j - S_i \geq p_i$ and $S_j - S_i \geq -p_j$ can be added to the initial convex program and because for each $k \in \mathcal{R}^\gamma$ and each $i \in O_k$, the construction of the corresponding sets A^{ik} and B^{ik} requires at most $|O_k|$ steps, the height of the branch-and-bound tree is $\mathcal{O}(\max[n^2, \sum_{k \in \mathcal{R}^\gamma} |O_k|^2])$.

References

[1] De Reyck, B., Herroelen, W.S. (1998). A branch-and-bound procedure for the resource-constrained project scheduling problem with generalized precedence relations. *European Journal of Operational Research*, 111, 152–174.

[2] Laborie, P. (2001). Algorithms for propagating resource constraints in AI planning and scheduling: Existing approaches and new results. *Proceedings of the IJCAI-01 Workshop on Planning with Resources*. Seattle, WA, USA.

[3] Neumann, K. (2002). Scheduling of batch production in process industries. *Proceedings of PMS 2002*, Valencia.

[4] Trautmann, N. (2002). Storage problems in process industries. *Proceedings of PMS 2002*, Valencia.