A Serial Schedule-Generation Scheme for Preemptive Project Scheduling Problems with Generalized Precedence Relations and Regular Min-Max Criteria

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15th International PMS Conference, Valencia

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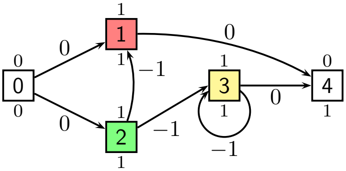
PMS 2016

Outline

Preemptive project scheduling problem

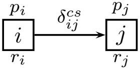
- Project consists of preemptive activities $i \in V$ with durations p_i
- Activities i require r_{ik} units of renewable resources $k \in \mathcal{R}$ with capacities R_k
- Minimum time lags δ_{ij}^{cs} between completion time C_i of activity i and start time S_j of activity j with $(i, j) \in E \subseteq V \times V$
  - $-\delta_{ij}^{cs} > 0$: maximum start-to-completion time lag between j and i
  - $-\delta_{ii}^{cs} \geq p_i$: maximum makespan of i
- Sought: feasible schedule $x : t \mapsto (x_j(t))_{j \in V}$ minimizing objective function $f_{max}(C) = \max_{j \in V} f_j(C_j)$ with regular f_j
  - $x_j(t)$: percentage of activity j processed by time $t \geq 0$
  - $S_j = \sup\{t \geq 0 \mid x_j(t) = 0\}$, $C_j = \inf\{t \geq 0 \mid x_j(t) = 1\}$
  - $y_j(t) := p_j \cdot \frac{\mathrm{d}^+ x_j}{\mathrm{d}t}(t) = \begin{cases} 1, & \text{if } j \text{ is in progress at time } t \\ 0, & \text{otherwise} \end{cases}$

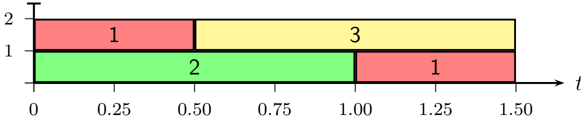
Example

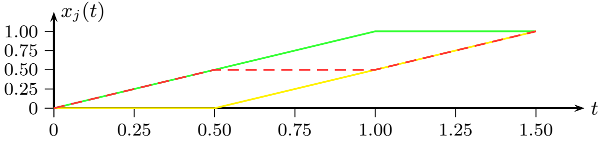


$\boxed{R = 2,\ f_{max} = C_{max}}$

Legend:







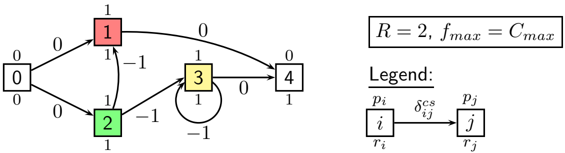
Conceptual model

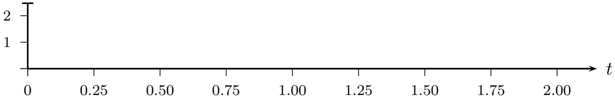
$$(P)\left\{\begin{array}{lll} \text{Minimize} & f_{max}(C) = \max_{j\in V} f_j(C_j) & \\ \text{subject to} & r_k(t) := \sum_{i\in V} r_{ik} y_i(t) \leq R_k & (k \in \mathcal{R};\ t \geq 0) \\ & S_j \geq C_i + \delta_{ij}^{cs} & ((i,j) \in E) \\ & S_j \geq 0,\ C_j \geq S_j + p_j & (j \in V) \end{array}\right.$$

- Non-preemptive problem contained as a special case ($\delta_{ii}^{cs} = -p_i$)
- Feasibility variant strongly NP-hard
- By convention $\delta_{ij}^{cs} \leq 0$ ($\delta_{ij}^{cs} > 0$ as dummy h with $p_h = \delta_{ij}^{cs}$)
- Each activity interrupted at most $n-1$ times
- Schedule encoded as set Σ of at most n^2 triples (j, s_j, c_j) defining time intervals $[s_j, c_j[$ during which parts of activities j are in progress

Why naïve adaption of SGS by Franck et al. (2001) fails

- In each iteration select eligible activity j^* to be started or resumed
- Schedule j^* at earliest time- and resource-feasible point in time
- Allow to suspend execution of j^* at next decision point (release, start, or completion of some activity)





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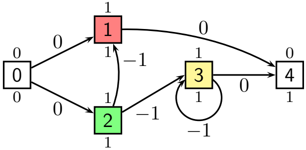

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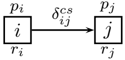

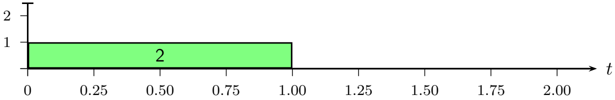

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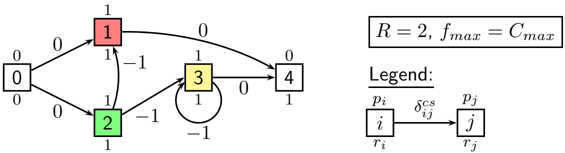

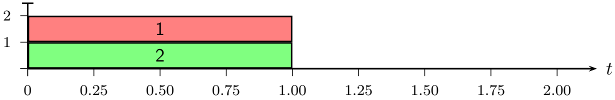

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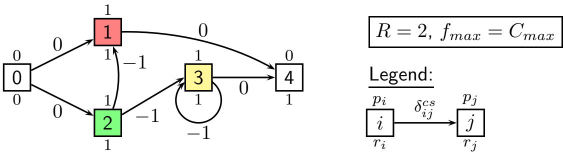
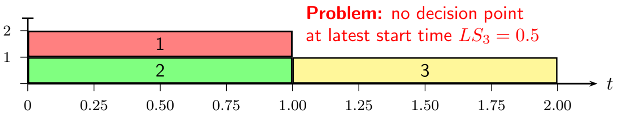




- Need additional decision points arising from objective function value

The solution: iterate threshold instances

- Given upper bound v on objective function value $f_{max}(C)$,

$$f_{max}(C) = \max_{j \in V} f_j(C_j) \leq v \Leftrightarrow f_j(C_j) \leq v \ (j \in V)$$
$$\Leftrightarrow C_j \leq f_j^{-1}(v) \ (j \in V)$$

with $f_j^{-1}(v) := \sup\{C_j \mid f_j(C_j) \leq v\}$ and $\sup \emptyset := -\infty$

- Introducing $-\delta_{j0}^{cs} = f_j^{-1}(v)$ for $j \in V$ ensures $f_{max}(C) \leq v$
- Perform binary search for smallest v over interval $[lb, ub]$
  - Start with $v = \frac{lb+ub}{2}$
  - Apply preemptive SGS to instance with $-\delta_{j0}^{cs} = f_j^{-1}(v)$
    - If feasible schedule is found: $f_{max}^* \leq f_{max}(C) \leq v$, put $ub := f_{max}(C)$
    - Otherwise: put $lb := v$
  - Recursively continue search on $[lb, ub]$ until $ub - lb \leq \varepsilon$

Basic principle of preemptive SGS

- Translate upper bound v into time lags $-\delta_{j0}^{cs} = f_j^{-1}(v)$ for $j \in V$
- Compute distance matrix $D^{cs} = (d_{ij}^{cs})_{i,j\in V}$ of transitive time lags
  - $ES_j = d_{0j}^{cs}$: earliest time-feasible start time of activity j
  - $LC_j = -d_{j0}^{cs}$: latest time-feasible completion times of activity j
- Initialize earliest time-feasible start times $es_j := ES_j$ of pending parts of activities j
- Activity j eligible for being started or resumed if all predecessors $i \in Pred^{\prec}(j)$ have been completed, i. e., $Pred^{\prec}(j) \subseteq \mathcal{C}$, where

$i \prec j \Leftrightarrow i$ must be started no later than j but not vice versa

$$\Leftrightarrow \max\{d_{ij}^{cs}, d_{i0}^{cs} + es_j\} + p_i \geq 0 \wedge \max\{d_{ji}^{cs}, d_{j0}^{cs} + es_i\} + p_j < 0$$

- Select some eligible activity $j^* \in \mathcal{E}$ by applying priority rule π

Basic principle of preemptive SGS

- Determine earliest resource-feasible start time $s_{j^*} \geq es_{j^*}$
- If $s_{j^*} \leq LC_{j^*} - p_{j^*}$, execute j^* up to next decision point c_{j^*}
- Set $\mathcal{D}$ of decision points contains
  - earliest completion time $s_{j^*} + p_{j^*}$ of j^*
  - start and completion times s_j, c_j of executed activity parts
  - earliest and latest start and completion times $es_j, es_j + p_j, LC_j - p_j, LC_j$ of pending activity parts
- Decrease residual processing time p_{j^*} by $c_{j^*} - s_{j^*}$ and update earliest start and latest completion times of pending activities
- If $s_{j^*} > LC_{j^*} - p_{j^*}$, try to repair the schedule by calling unscheduling procedure
- Proceed until all activities $j \in V$ have been entirely added to the schedule, i. e., $\mathcal{C} = V$

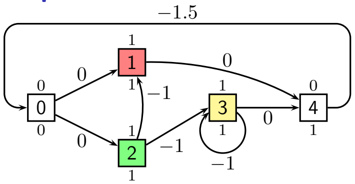
Basic principle of unscheduling procedure

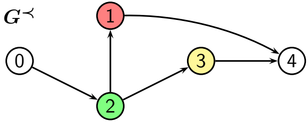
- If $s_{j^*} > LC_{j^*} - p_{j^*}$, SGS has run into a deadlock
- Clear parts of the schedule to remove deadlock
- Identify set $\mathcal{U}$ of started activities $j \in \mathcal{S}$ that determined the latest completion time LC_{j^*} of activity j^*

$$\mathcal{U} = \{j \in \mathcal{S} \mid LC_{j^*} = S_j - d^{cs}_{j^*j}\}$$

- Remove all activity parts with $s_j \geq \min_{i \in \mathcal{U}} S_i$ from the schedule
- Delay activities $j \in \mathcal{U}$ by $\Delta = s_{j^*} + p_{j^*} - LC_{j^*}$: put $es_j := S_j + \Delta$
- If $es_j + p_j > -d^{cs}_{j0}$, deadlock could not be resolved: terminate
- Otherwise return to preemptive SGS

Example

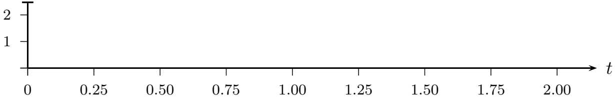




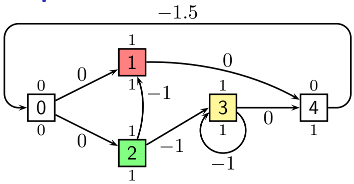
$$D^{cs} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \\ -1.5 & -1 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \end{pmatrix}$$

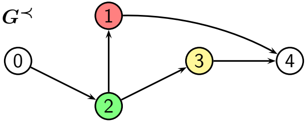
j	1	2	3
es_j	0	0	0
LC_j	1.5	1.5	1.5
S_j	–	–	–
C_j	–	–	–

$\mathcal{S} = \{0\}$
$\mathcal{C} = \{0\}$



Example





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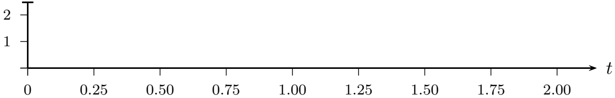
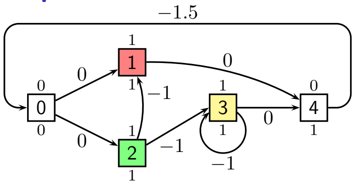
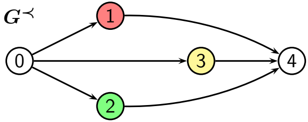


$$\mathcal{E} = \{2\},\ j^* = 2,\ s_2 = 0,\ \mathcal{D} = \{0, 0.5, 1, 1.5\},\ c_2 = 0.5,\ p_2 = 0.5$$

Example





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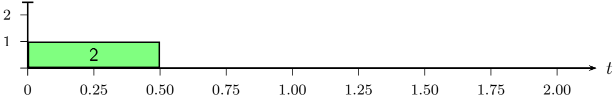
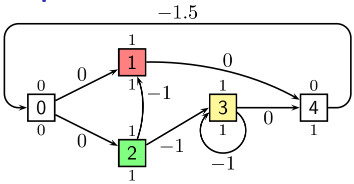
j	1	2	3
es_j	0	0.5	0
LC_j	1.5	1.5	1.5
S_j	–	0	–
C_j	–	–	–

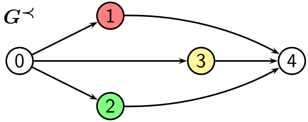
$\mathcal{S} = \{0, 2\}$
$\mathcal{C} = \{0\}$



$\mathcal{E} = \{1, 2, 3\}$, $j^* = 3$, $s_3 = 0$, $\mathcal{D} = \{0, 0.5, 1, 1.5\}$, $c_3 = 0.5$, $p_3 = 0.5$

Example





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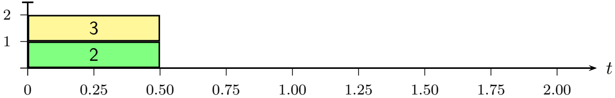
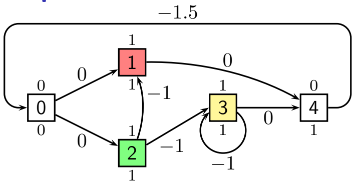
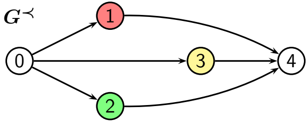


$\mathcal{E} = \{1, 2, 3\}$, $j^* = 1$, $s_1 = 0.5$, $\mathcal{D} = \{0, 0.5, 1, 1.5\}$, $c_1 = 1$, $p_1 = 0.5$

Example





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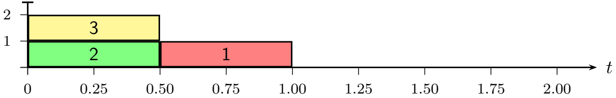

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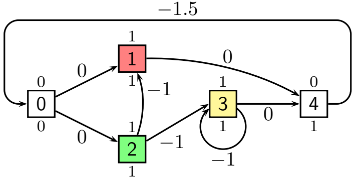

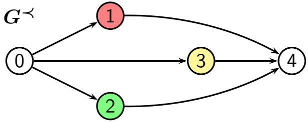

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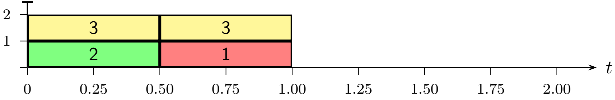

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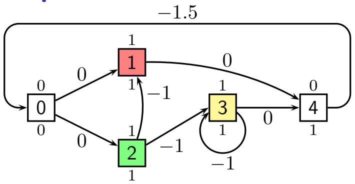

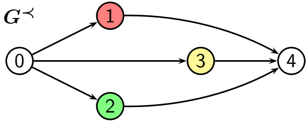

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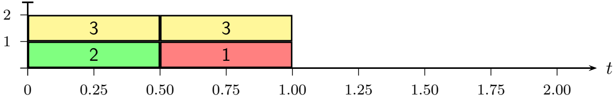
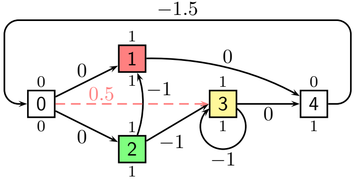
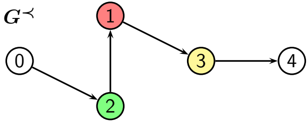


$s_2 + p_2 > LC_2$: *unschedule*$(2, 0.5)$, $\mathcal{U} = \{3\}$, $t^* = S_3 = 0$, $es_3 = 0.5$

Example





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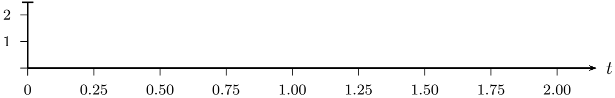
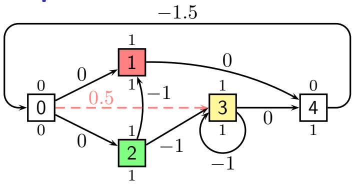
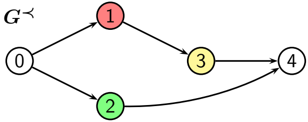


$\mathcal{E} = \{2\},\ j^* = 2,\ s_2 = 0,\ \mathcal{D} = \{0, 0.5, 1, 1.5\},\ c_2 = 0.5,\ p_2 = 0.5$

Example

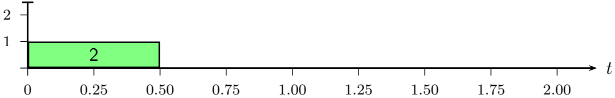




$$D^{cs} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \\ -1.5 & -1 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \end{pmatrix}$$

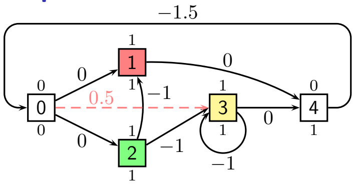
j	1	2	3
es_j	0	0.5	0.5
LC_j	1.5	1.5	1.5
S_j	–	0	–
C_j	–	–	–

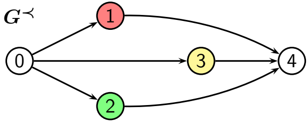
$\mathcal{S} = \{0, 2\}$
$\mathcal{C} = \{0\}$



$\mathcal{E} = \{1, 2\},\ j^* = 1,\ s_1 = 0,\ \mathcal{D} = \{0, 0.5, 1, 1.5\},\ c_1 = 0.5,\ p_1 = 0.5$

Example

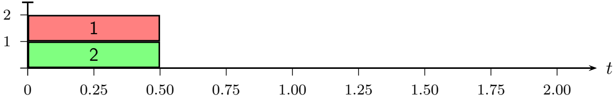




$$D^{cs} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \\ -1.5 & -1 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \end{pmatrix}$$

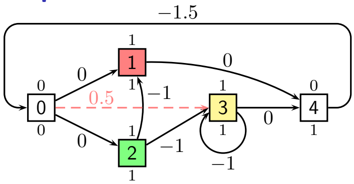
j	1	2	3
es_j	0.5	0.5	0.5
LC_j	1.5	1	1.5
S_j	0	0	–
C_j	–	–	–

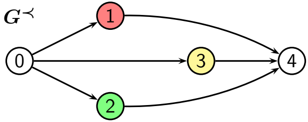
$\mathcal{S} = \{0, 1, 2\}$
$\mathcal{C} = \{0\}$



$\mathcal{E} = \{1, 2, 3\},\ j^* = 2,\ s_2 = 0.5,\ \mathcal{D} = \{0, 0.5, 1, 1.5\},\ c_2 = 1,\ p_2 = 0$

Example

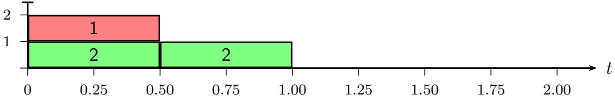




$$D^{cs} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \\ -1.5 & -1 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \end{pmatrix}$$

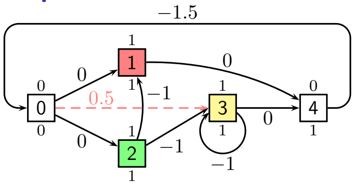
j	1	2	3
es_j	0.5	0.5	0.5
LC_j	1.5	1	1.5
S_j	0	0	–
C_j	–	1	–

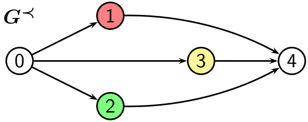
$\mathcal{S} = \{0, 1, 2\}$
$\mathcal{C} = \{0, 2\}$



$\mathcal{E} = \{1, 3\},\ j^* = 3,\ s_3 = 0.5,\ \mathcal{D} = \{0, 0.5, 1, 1.5\},\ c_3 = 1,\ p_3 = 0.5$

Example

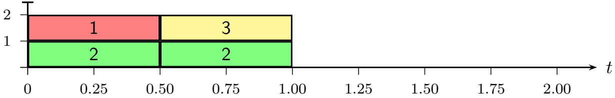




$$D^{cs} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \\ -1.5 & -1 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \end{pmatrix}$$

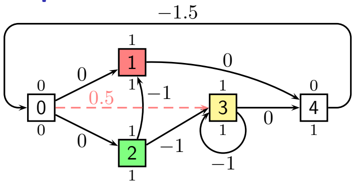
j	1	2	3
es_j	0.5	0.5	1
LC_j	1.5	1	1.5
S_j	0	0	0.5
C_j	–	1	–

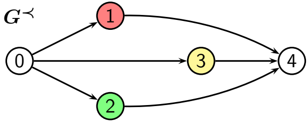
$\mathcal{S} = \{0, 1, 2, 3\}$
$\mathcal{C} = \{0, 2\}$



$\mathcal{E} = \{1, 3\},\ j^* = 1,\ s_1 = 1,\ \mathcal{D} = \{0, 0.5, 1, 1.5\},\ c_1 = 1.5,\ p_1 = 0$

Example

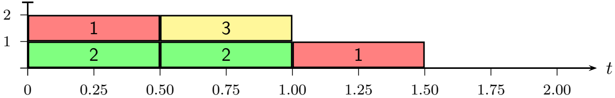




$$D^{cs} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \\ -1.5 & -1 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \end{pmatrix}$$

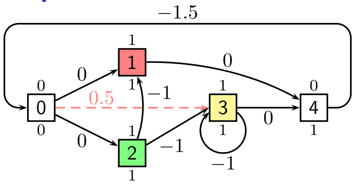
j	1	2	3
es_j	0.5	0.5	1
LC_j	1.5	1	1.5
S_j	0	0	0.5
C_j	1.5	1	–

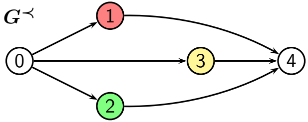
$\mathcal{S} = \{0, 1, 2, 3\}$
$\mathcal{C} = \{0, 1, 2\}$



$\mathcal{E} = \{3\},\ j^* = 3,\ s_3 = 1,\ \mathcal{D} = \{0, 0.5, 1, 1.5\},\ c_3 = 1.5,\ p_3 = 0$

Example

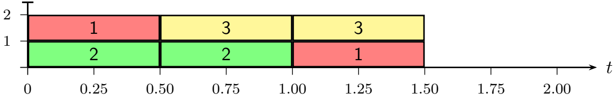




$$D^{cs} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \\ -1.5 & -1 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1 & 0 \\ -1.5 & -1.5 & -1.5 & -1.5 & 0 \end{pmatrix}$$

j	1	2	3
es_j	0.5	0.5	1
LC_j	1.5	1	1.5
S_j	0	0	0.5
C_j	1.5	1	1.5

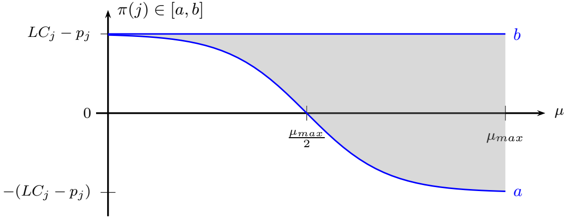
$\mathcal{S} = \{0, 1, 2, 3\}$
$\mathcal{C} = \{0, 1, 2, 3\}$



Randomized multi-start procedure

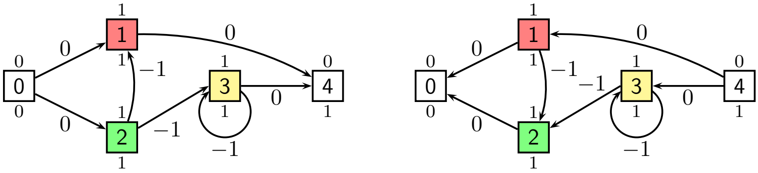
- In multi-start procedure, binary search algorithm can be run with different priority rules π
- Start with minimum latest start time rule: $\pi(j) = LC_j - p_j$
- Stepwise increase random bias according to sigmoid function $\sigma(\mu)$ for passes $\mu = 1, \ldots, \mu_{max}$

$$\pi(j) = (LC_j - p_j) \cdot (1 - 2\sigma(\mu) \cdot rnd) \text{ with } \sigma(\mu) = \frac{1}{1+e^{-\alpha\cdot(\mu-\mu_{max}/2)}}$$



Forward and backward scheduling

- Scheduling pending activity parts at their earliest feasible start times corresponds to forward scheduling
- Backward scheduling can be emulated by applying SGS on "inverted" project network (Hanzálek and Šůcha 2009)
- Substitute time lags δ_{ij}^{cs} into time lags $\bar{\delta}_{ji}^{cs} = \delta_{ij}^{cs}$
- Original and inverted instances $I, \bar{I}$ of threshold problem equivalent
- Schedule $\bar{\Sigma}$ for $\bar{I}$ transformed into schedule Σ for I by replacing triples $(j, \bar{s}_j, \bar{c}_j) \in \bar{\Sigma}$ with triples $(j, \bar{C}_{max} - \bar{c}_j, \bar{C}_{max} - \bar{s}_j)$



Experimental performance analysis

- Testsets UBO10, UBO20, UBO50, UB100 with 90 instances each (Franck et al. 2001)
- Objective function project duration $f_{max}(C) = C_{max}$
- Lower and upper bounds for preemptive instances computed with MILP formulation (S. and Paetz 2015)
- Upper bounds for non-preemptive instances according to benchmark files on ProGen/max homepage[1]
- Tested configurations
  - Forward scheduling: $\mu_{max} = 100$, $\varepsilon = 10^{-4}$
  - Backward and forward scheduling: $\mu_{max} = 50$, $\varepsilon = 10^{-4}$
- Multi-start procedure coded in C#
- Intel i5 PC with 3.4 GHz clock pulse and 8 GB RAM, OS Win 7 Professional 64 Bit

[1] www.wiwi.tu-clausthal.de/abteilungen/produktion/forschung/schwerpunkte/project-generator/

Computational results

Forward scheduling, 100 passes

	Δ_{milp}	Δ_{npmtn}	p_{imp}	p_{fnd}	t_{cpu}	$\# it$
$n = 10$	0.50 %	−1.71 %	27.78 %	4.44 %	0.14 s	1668
$n = 20$	0.06 %	−1.68 %	36.67 %	10.00 %	0.37 s	1746
$n = 50$	n/a	−1.35 %	44.44 %	3.33 %	3.15 s	1728
$n = 100$	n/a	−1.16 %	46.67 %	0.00 %	24.50 s	1837

Forward and backward scheduling, 50 passes

	Δ_{milp}	Δ_{npmtn}	p_{imp}	p_{fnd}	t_{cpu}	$\# it$
$n = 10$	0.17 %	−2.04 %	31.11 %	4.44 %	0.14 s	1662
$n = 20$	−1.08 %	−2.66 %	46.67 %	11.11 %	0.34 s	1715
$n = 50$	n/a	−2.12 %	52.22 %	2.22 %	3.37 s	1732
$n = 100$	n/a	−1.41 %	57.78 %	1.11 %	25.16 s	1835

Summary

- SGS for preemptive project scheduling problems with generalized precedence relations and regular min-max criteria
- Create new decision points by iterating threshold instances
- Multi-start heuristic with randomly biased priority indices
- Improvements on benchmark results for non-preemptive problems
  - Preemption gains obtained for 47 % of the instances
  - Feasible schedules generated for 17 out of 66 instances that are infeasible when preemption is not allowed

Future research

- Performance evaluation for other regular min-max criteria
- Metaheuristics embedding preemptive SGS

References

Franck B, Neumann K, Schwindt C (2001)
Truncated branch-and-bound, schedule-construction, and schedule-improvement procedures for resource-constrained project scheduling
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In: Schwindt C, Zimmermann J (eds) Handbook on Project Management and Scheduling Vol. 1. Springer, Cham, Heidelberg, New York, Dordrecht, London, pp 251–295