The resource transfer problem

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1 Introduction

In this paper we propose the resource transfer problem (RTP), a framework for modeling and solving rich vehicle routing and complex scheduling problems in a unified way. The RTP basically consists in scheduling a set of activities to be performed at different locations in a network subject to generalized precedence relations, where the activities represent customer demands to be served by a set of vehicles and further resources like personnel, machines, or handling facilities. The resources may be transferred between the locations and may require sequence-dependent changeovers between the executions of consecutive activities. The model covers a large variety of further side constraints arising from specific requirements in practical vehicle routing and scheduling problems. Examples of services delivered at the customer locations include pickups and deliveries of goods, visits of patients to ambulant medical care services, or tasks of multiple projects carried out at different sites. The commodities, patients, or facilities are transferred between the locations by a fleet of vehicles of different capacities, the travel times depending on both the type of the vehicle and the type of the transferred goods or persons.

The remainder of this paper is organized as follows. In Sect. 2 we develop an event-based model formulation of the generic RTP and we introduce a graph-based encoding of RTP instances. Sect. 3 discusses examples of classical vehicle routing and scheduling problems and specific practical requirements that are covered by the RTP model. In Sect. 4 we sketch the principle of a constraint-based branch-and-bound method for RTP and report on some preliminary computational experience with the algorithm.

2 Problem statement

We consider a finite set of activities with deterministic durations. The execution of each activity is assigned to a node of a network and requires renewable and storage resources, which may be shared among the nodes. The renewable resources $k \in \mathcal{R}^\rho$ represent potential factors such as production facilities, vehicles, or personnel, whereas the storage resources $\ell \in \mathcal{R}^\gamma$ keep records of vehicle loads and consumable factors like materials or energy. The capacity R_k of a renewable resource k corresponds to the number of resource units available, and the capacity R_ℓ of a storage resource ℓ specifies a holding or load capacity. Based on the approach of Selensky (2001), a vehicle can be thought of as a combination of a unary resource k of capacity $R_k = 1$ and one storage resource ℓ for each passenger or commodity hauled and each loading restriction.

We represent an activity by the pair of its start and completion events i^s and i^c. The set V of all events i may also include dummy events like the start and the end of the planning period. The resource requirements of the activities carry over to the events, where the requirement of event i for some renewable or storage resource k or ℓ is denoted by $r_{ik} \geq 0$ and $r_{i\ell}$. A negative $r_{i\ell} < 0$ means that resource ℓ is depleted by $-r_{i\ell}$ units at the occurrence of event i, whereas $r_{i\ell} > 0$ stands for a replenishment of ℓ.

Different events $i, j \in V$ can be linked by two types of temporal relationships: prescribed minimum time lags δ_{ij} and transfer times $\vartheta_{ij} \geq 0$ between the respective occurrence times t_i and t_j. A negative minimum time lag $\delta_{ij} < 0$ can be interpreted as a positive maximum time lag $-\delta_{ij}$ between t_j and t_i. Minimum time lags have to be observed for all pairs (i,j) of a given set $A \subset V \times V$, irrespective of the resource allocation to the events, while transfer times refer to all pairs (i,j) of events with $t_j > t_i$ sharing units of some renewable resource k. The minimum time lags serve us to express precedence relations, time windows, or synchronization requirements, the transfer times are used to model travel or changeover times as well as the activity durations.

We suppose that an event i can occur in alternative modes $m \in \mathcal{M}_i$ differing in resource requirements, minimum time lags, and transfer times. The multi-mode setting accounts for alternatives arising from the availability of heterogeneous vehicles or personnel mastering different skills. As a consequence, we generalize the resource requirements, time lags, and transfer times to mode-dependent quantities r_{ik}^m, $r_{i\ell}^m$, $\delta_{ij}^{mm'}$, and $\vartheta_{ij}^{mm'}$, where $m \in \mathcal{M}_i$ and $m' \in \mathcal{M}_j$. The mode selection problem consists in choosing one occurrence mode $m \in \mathcal{M}_i$ for each event i. The problem is encoded via binary decision variables $x_i^m \in \{0,1\}$, which equal one precisely if i occurs in mode m. To simplify notation we introduce the symbols $r_{ik}(\boldsymbol{x}) = \sum_{m\in\mathcal{M}_i} r_{ik}^m \cdot x_i^m$, $r_{i\ell}(\boldsymbol{x}) = \sum_{m\in\mathcal{M}_i} r_{i\ell}^m \cdot x_i^m$, $\delta_{ij}(\boldsymbol{x}) = \sum_{m\in\mathcal{M}_i}\sum_{m'\in\mathcal{M}_j} \delta_{ij}^{mm'} \cdot x_i^m \cdot x_j^{m'}$, and $\vartheta_{ij}(\boldsymbol{x}) = \sum_{m\in\mathcal{M}_i}\sum_{m'\in\mathcal{M}_j} \vartheta_{ij}^{mm'} \cdot x_i^m \cdot x_j^{m'}$ for the resource requirements, time lags, and transfer times resulting from mode assignment $\boldsymbol{x} = (x_i^m)_{i\in V, m\in\mathcal{M}_i}$.

The resource transfer problem RTP consists in assigning an occurrence time $t_i \geq 0$ and a mode $m \in \mathcal{M}_i$ to each event i so as to minimize a regular objective function $f(\boldsymbol{t},\boldsymbol{x})$ in occurrence times $\boldsymbol{t} = (t_i)_{i\in V}$ and mode assignment $\boldsymbol{x}$ subject to renewable and storage resource constraints, minimum time lags, transfer times, and constraints imposing conditions on the allocation of resource units to events. With $U_k(i)$ being the set of units of renewable resource k assigned to event i, the problem can be formulated in the following way:

$$
(\text{RTP}) \left\{
\begin{array}{lll}
\text{Min. } f(\boldsymbol{t},\boldsymbol{x}) & & \\
\text{s.t. } \sum_{m\in\mathcal{M}_i} x_i^m = 1 & (i \in V) & (1) \\
|U_k(i)| = r_{ik}(\boldsymbol{x}),\ U_k(i) \subseteq \{1,\ldots,R_k\} & (i \in V;\ k \in \mathcal{R}^\rho) & (2) \\
0 \leq \sum_{i\in V: t_i \leq t_j} r_{i\ell}(\boldsymbol{x}) \leq R_\ell & (\ell \in \mathcal{R}^\gamma;\ j \in V) & (3) \\
t_j - t_i \geq \delta_{ij}(\boldsymbol{x}) & ((i,j) \in A) & (4) \\
t_j - t_i \geq \vartheta_{ij}(\boldsymbol{x}) \vee t_i - t_j \geq \vartheta_{ji}(\boldsymbol{x}) & (i,j \in V : U_k(i) \cap U_k(j) \neq \emptyset & \\
 & \text{ for some } k \in \mathcal{R}^\rho) & (5) \\
U_k(i) \subseteq U_k(j) & ((i,j) \in I;\ k \in \mathcal{R}_{ij}) & (6) \\
U_k(i) \cap U_k(j) = \emptyset & (\{i,j\} \in \bar{I};\ k \in \bar{\mathcal{R}}_{ij}) & (7) \\
t_i \geq 0,\ x_i^m \in \{0,1\} & (i \in V;\ m \in \mathcal{M}_i) &
\end{array}
\right.
$$

The constraints ensure that (1) each event occurs in exactly one mode, (2) each event is allocated the numbers of resource units specified by the resource requirements, (3) the stock levels of the storage resources remain between zero and the capacity, and (4) the time lags as well as (5) the transfer times are observed. In case of symmetric transfer times, constraint (5) can also be written as $|t_j - t_i| \geq \vartheta_{ij}(\boldsymbol{x})$. Furthermore, we consider the unit-inclusion constraints (6) and the unit-incompatibility constraints (7), which impose conditions on the resource unit assignments of different events. The unit-inclusion constraint for pair $(i,j) \in I$ states that all units of the resources k from set $\mathcal{R}_{ij} \subseteq \mathcal{R}^\rho$ allocated to i must also be assigned to j. Note that when $(i,j) \in I$ and $(j,i) \in I$, the same units of resources $k \in \mathcal{R}_{ij}^\rho \cap \mathcal{R}_{ji}^\rho$ must be allocated to i and j. Such a unit-identity constraint must be satisfied for each combination $\{i^s, i^c\}$ of start and completion events of the same activity. A unit-incompatibility constraint is the converse condition saying that for combinations $\{i,j\} \in \bar{I}$, the two events i,j must not share any unit of the resources k from set $\bar{\mathcal{R}}_{ij} \subseteq \mathcal{R}^\rho$.

An instance of problem RTP can be represented as a tuple $(\boldsymbol{R}, \boldsymbol{r}, G_L, G_T, G_I, G_{\bar{I}})$ with $\boldsymbol{R} = (R_k, R_\ell)_{k\in\mathcal{R}^\rho, \ell\in\mathcal{R}^\gamma}$ and $\boldsymbol{r} = (r_{ik}^m, r_{i\ell}^m)_{i\in V, m\in\mathcal{M}_i, k\in\mathcal{R}^\rho, \ell\in\mathcal{R}^\gamma}$. The graphs G_L, G_T, G_I, and $G_{\bar{I}}$ encode the minimum time lags, the transfer times, the unit-inclusion constraints, and the unit-incompatibility constraints. The time lag graph $G_L = (V, A, \boldsymbol{\delta})$ is a directed network with node set V, arc set A, and arc weights $\delta_{ij} = (\delta_{ij}^{mm'})_{m \in \mathcal{M}_i, m' \in \mathcal{M}_j}$ for $(i,j) \in A$. Such a network representing generalized precedence relations between the start or completion events of activities was introduced by Elmaghraby and Kamburowski (1992), who considered the single-mode case and referred to G_L as the activity network. Given mode assignment $\boldsymbol{x}$, there exists a vector $\boldsymbol{t}$ of occurrence times t_i satisfying the time lag constraints (4) precisely if network $G_L(\boldsymbol{x})$ with arc weights $\delta_{ij}(\boldsymbol{x})$ does not contain any cycle of positive length. In this case, the length of a longest path from node i to node j in $G_L(\boldsymbol{x})$ equals the transitive minimum time lag $d_{ij}(\boldsymbol{x})$ between events i and j induced by the time lags $\delta_{gh}(\boldsymbol{x})$ with $(g,h) \in A$. The transfer graph $G_T = (V, V^2, \boldsymbol{\vartheta})$ represents the mode-dependent transfer times ϑ_{ij} of resource units among the events $i, j \in V$. Depending on whether the transfer times are symmetric or not, G_T is a complete undirected graph (*i. e.*, $V^2 = \{\{i,j\} \mid i < j\}$) or a complete directed graph (*i. e.*, $V^2 = \{(i,j) \mid i \neq j\}$). The weights of the edges $\{i,j\}$ or arcs (i,j) are the transfer time matrices $\vartheta_{ij} = (\vartheta_{ij}^{mm'})_{m \in \mathcal{M}_i, m' \in \mathcal{M}_j}$. For each mode assignment $\boldsymbol{x}$, the transfer times must satisfy the conditions $\vartheta_{ii}(\boldsymbol{x}) = 0$ and $\vartheta_{hj}(\boldsymbol{x}) \leq \vartheta_{hi}(\boldsymbol{x}) + \vartheta_{ij}(\boldsymbol{x})$ for all $h, i, j \in V$. Consequently, $\vartheta_{hj}(\boldsymbol{x})$ is equal to the length of a shortest path from node h to node j in graph $G_T(\boldsymbol{x})$ with edge or arc weights $\vartheta_{ij}(\boldsymbol{x})$. The unit-inclusion graph $G_I = (V, I)$ is a directed graph with node set V and arc set I, whereas the unit-incompatibility graph $G_{\bar{I}} = (V, \bar{I})$ is an undirected graph with node set V and edge set $\bar{I}$. The arcs (i,j) of G_I and the edges $\{i,j\}$ of $G_{\bar{I}}$ are labeled with the respective sets $\mathcal{R}_{ij}$ and $\bar{\mathcal{R}}_{ij}$. Given a set of renewable resources $\mathcal{R}'$, the directed paths in subgraph $G'_I = (V, I')$ containing all arcs $(i,j) \in I$ with $\mathcal{R}_{ij} \supseteq \mathcal{R}'$ provide all immediate and transitive unit-inclusion dependencies for the resources $k \in \mathcal{R}'$. For each strong component of G'_I with node set V', the unit-inclusion constraints of G_I imply unit-identity constraints for all events $i \in V'$ and all resources $k \in \mathcal{R}'$. Symmetrically, the undirected paths of the subgraph $G'_{\bar{I}}$ containing all edges $\{i,j\} \in \bar{I}$ with $\bar{\mathcal{R}}_{ij} \supseteq \mathcal{R}'$ define the immediate or transitive unit-incompatibility constraints for the resources $k \in \mathcal{R}'$.

3 Modeling power of the framework

The resource transfer problem covers a large variety of classical vehicle routing and complex scheduling problems as well as specific constraints arising in practical routing and scheduling applications.

Examples of complex scheduling problems that can be modeled as RTP are resource-constrained multi-site project scheduling problems with renewable and storage resources, sequence-dependent changeover times, generalized precedence relations, and multiple activity execution modes (for a survey on resource-constrained project scheduling models see, *e. g.*, Hartmann and Briskorn 2010). Generalized precedence relations allow for considering release dates, deadlines, quarantine times, precedence constraints, and further temporal synchronization constraints. Sequence-dependent changeover times, limited buffer capacities, and material-availability constraints are further requirements arising frequently in production scheduling applications. In case of multi-site scheduling problems, spatial synchronization requirements and resource transfers among the locations can be taken into account.

In addition, RTP includes many classical vehicle routing problems VRP such as the capacitated VRP, the VRP with time windows, the VRP with mixed linehauls and backhauls, the pickup-and-delivery problem, or the dial-a-ride problem. For recent reviews on these models we refer to Toth and Vigo (2015) and Parragh *et al.* (2008a,b). Further constraints that can easily be modeled in the RTP framework are, *e. g.*, maximum ride times of vehicles, driver breaks, freight transshipments, as well as temporal and spatial synchronization requirements. Moreover, multiple loading constraints, multiple commodities and compartments, and stocking facilities can be taken into account. The multi-mode setting enables us to model heterogeneous fleets, including vehicle-dependent travel or service times, site dependencies, and different driver qualifications. Further restrictions may arise from incompatibilities between persons or goods. These incompatibilities may either apply to the whole trip of a vehicle or only prevent the joint transport.

Of course, all problems arising from the combination of the above complex scheduling and vehicle routing problems can also be formulated as RTP instances. Integrated vehicle routing and production scheduling problems are, *e. g.*, encountered in supply chains where products are manufactured in distributed multi-echelon production networks and intermediate and final products have be transferred between the suppliers, manufacturers, wholesalers, and retailers. Nonetheless, there exist problem settings that do not immediately fit the RTP framework. Examples are arc routing problems, split deliveries, or problems including optional stopovers or services like truck-and-trailer routing or prize-collecting problems.

4 Solution approach

As a solver for RTP we use a branch-and-bound algorithm (B&B) invoking constraint propagation to reduce the search space. Following the approach of the time-based branching scheme by Dorndorf *et al.* (2000) for the resource-constrained project scheduling problem with generalized precedence relations, we adapted the algorithm to cope with events, transfer times, multiple modes, and storage resources. We use generalizations of consistency tests from literature to identify and remove inconsistent occurrence times of events. In each node of the enumeration tree, all consistency tests are cyclically run until the domain has reached a fixed point. The consistency tests used for the renewable resources are different edge finding techniques and shaving. For unary resources the disjunctive constraint can also be applied. For the storage resources we employ the balance test and the profile test. For a detailed description of these consistency tests we refer to Baptiste *et al.* (2001), p. 21 and Chap. 3 and Schwindt (2005), pp. 25 f. and pp. 36 ff.

To validate the RTP model and to get a first insight into the performance of the solver, we ran some preliminary computational experiments based on instances of the pickup-and-delivery problem with homogeneous goods and time windows. The instances were adapted from the 1-TSPPD traveling salesman instances of Hernández-Pérez (2015) by defining time windows for each customer visit and adding an extra vehicle to transform the TSP into a VRP. We considered 20 small instances with 10 and 20 customers and compared the results of the B&B to the schedules obtained with an MILP formulation of the RTP model presented in Sect. 2. The MILP was implemented in GAMS 24.4.6 with CPLEX 12.6.2 as MILP solver, and the B&B was coded in C++. The tests were performed on a PC with an Intel i7 processor with 4.0 GHz clock pulse, 32 GB RAM, and Windows 7 as operating system. All instances were solved with and without the consistency tests to investigate their effectiveness. For the instances including 20 customers we imposed a time limit of one day and did further experiments with a time limit of 100 sec per instance to investigate how fast good solutions are found.

Both versions of the B&B were able to close all instances with 10 customers within 2 sec, while CPLEX took a maximum of 6 sec. Applying the consistency tests led to a reduction in the number of enumeration nodes by a factor of 1.2 on average. Table 1 shows the results obtained for the instances with 20 customers. The second to fourth columns provide the numbers of enumerated nodes, while the fifth to seventh column display the required CPU times. The last three columns list the objective function values, where a star indicates that the value was proved to be optimal. B&B refers to the B&B without the consistency tests, B&B+Γ to the version with the tests, and CPLEX stands for the MILP solved by CPLEX. The last row shows the factor by which the number of enumeration nodes of the B&B is reduced on average by applying the consistency tests, the mean being taken over the instances closed by both versions of B&B. While B&B+Γ including the consistency tests completes the enumeration for 90 % of the instances, B&B without the tests is able to close 60 %, and CPLEX only 30 % of the instances. Moreover, B&B+Γ ran much faster than B&B and CPLEX, which is due to the significant search space reduction by a factor of 15 achieved by the consistency tests. Within the time limit of 100 sec, B&B+Γ already found an optimal solution for six out of ten instances and closed three of them. CPLEX only yields feasible solutions, none of which is optimal. These results indicate that first, for small instances B&B+Γ is able to provide good solutions within short CPU times and that second, the effectiveness of the consistency tests markedly increases with growing problem sizes.

Table 1. Results of the instances with 20 customers (CPU time limit 86,400 sec)

Instance	**# nodes (10^3 nodes)**			**CPU time (sec)**			$f(\boldsymbol{t}, \boldsymbol{x})$		
	B&B	**B&B+Γ**	**CPLEX**	**B&B**	**B&B+Γ**	**CPLEX**	**B&B**	**B&B+Γ**	**CPLEX**
n20q10J	9,932	188	9,328	3,126	99	51,892	386*	386*	386*
n20q10I	1,222	526	10,959	405	272	86,400	385*	385*	385
n20q10H	128,198	26,579	7,918	38,411	14,003	86,400	387*	387*	387
n20q10G	262,467	115,136	1,718	86,400	57,188	86,400	386	386*	391
n20q10F	275,703	51,098	3,338	86,400	25,275	86,400	372	372*	374
n20q10E	2,135	53	9,214	660	31	86,400	369*	369*	369
n20q10D	170,323	29,372	2,074	54,045	14,183	11,473	385*	385*	385*
n20q10C	25	1	113	7	<1	587	383*	383*	383*
n20q10B	261,131	179,579	2,199	86,400	86,400	86,400	395	384	384
n20q10A	272,180	15,391	3,075	86,400	8,009	86,400	388	388*	388
	Ratio: 15.13 to 1			Percentage of closed instances:			60,0 %	90,0 %	30,0 %

Preliminary results obtained for single- and multi-mode instances of resource-constrained project scheduling problems indicate that the solver is competitive with classical branch-and-bound algorithms for these problems, but is outperformed by the most recent approaches based on the concept of lazy clause generation LCG. Developing a generic LCG method for the RTP seems a particularly promising avenue of research, which we will pursue.

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